NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. Locate the point on the complex plane corresponding to the complex number given in Parts A–H. On one set of axes, label each point by its identifying letter. For example, the point corresponding to $5 + 2i$ should be labeled A.

   A. $5 + 2i$

   B. $3 - 2i$

   C. $-2 - 4i$

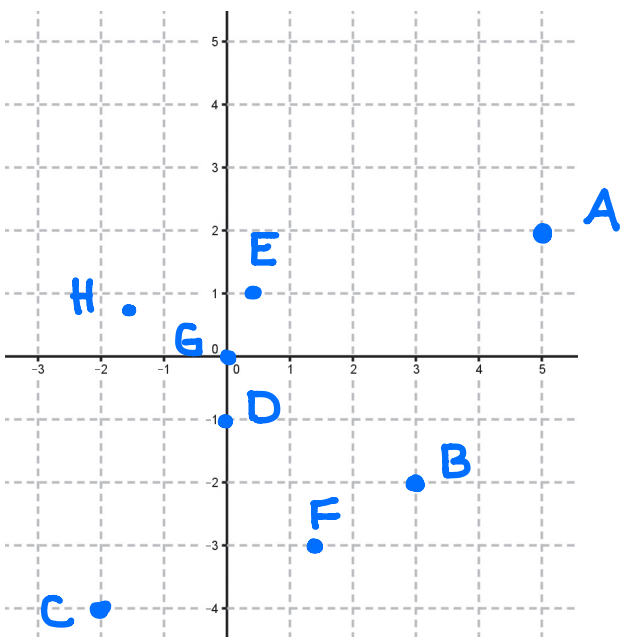
   D. $-i$

   E. $\frac{1}{2} + i$

   F. $\sqrt{2} - 3i$

   G. 0

   H. $-\frac{3}{2} + \frac{\sqrt{3}}{2}i$



2. Find the real values of x and y in each of the following equations using the fact that if $a + bi = c + di$, then $a = c$ and $b = d$.

   A. $5x + 3yi = 20 + 9i$

   $5x = 20$ $\quad$ $3yi = 9i$

   $x = 4$ $\quad$ $y = 3$

   B. $3(7 - 2x) - 5(4y - 3)i = x - 2(1 + y)i$

   $3(7-2x) = x$

   $21 - 6x = x$

   $21 = 7x$

   $3 = x$

   $-5(4y-3)i = -2(1+y)i$

   $-20y + 15 = -2 - 2y$

   $15 = -2 + 18y$

   $17 = 18y$

   $y = \frac{17}{18}$

   C. $2(5x + 9) = (10 - 3y)i$

   $2(5x+9) = 0$

   $10x + 18 = 0$

   $10x = -18$

   $x = -\frac{9}{5}$

   $0i = (10 - 3y)i$

   $0 = 10 - 3y$

   $3y = 10$

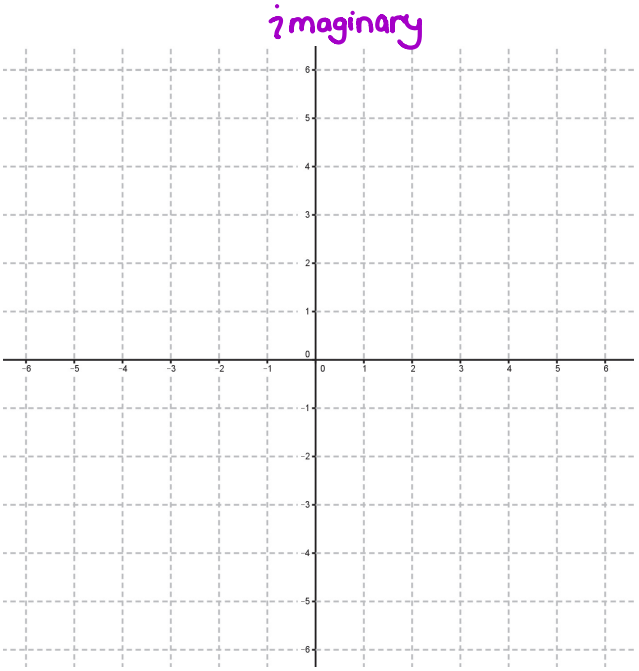
   $y = \frac{10}{3}$

3. Since $i^2 = -1$, we see that

$i^3 = i^2 \cdot i = -1 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1.$

Plot i, i^2, i^3, and i^4 on the complex plane, and describe how multiplication by each rotates points in the complex plane.



multiply by i → rotates 90° ccw

multiply by i^2 → rotates 180° ccw
$i^2 = -1$

multiply by i^3 → rotates 270 ccw
$i^3 = -i$

multiply by i^4 → rotates 360 ccw
$i^4 = 1$

4. Express each of the following in $a + bi$ form.

A. i^5 → $i^4 \cdot i = 1 \cdot i = i$ → $0 + i$

B. i^6 → $i^4 \cdot i^2 = 1 \cdot -1 = -1$ → $-1 + 0i$

C. i^7 → $i^4 \cdot i^3 = 1 \cdot -i = -i$ → $0 - i$

D. i^8 → $i^4 \cdot i^4 = 1 \cdot 1 = 1$ → $1 + 0i$

E. i^{102} → $i^{100} \cdot i^2 = 1 \cdot -1 = -1$ → $-1 + 0i$

$102 \div 4 = 25$ R 2

5. Evaluate the four products below.

A. $\sqrt{9} \cdot \sqrt{4}$

$3 \cdot 2 = \boxed{6}$

B. $\sqrt{9} \cdot \sqrt{-4}$

$\sqrt{-1}\ \sqrt{4} = 2i$

$\sqrt{9} \cdot 2i$

$3 \cdot 2i$

$\boxed{6i}$

C. $\sqrt{-9} \cdot \sqrt{4} \rightarrow 3i \cdot 2 = \boxed{6i}$

$\sqrt{-1}\ \sqrt{9} = 3i$

D. $\sqrt{-9} \cdot \sqrt{-4}$

$\sqrt{-1}\ \sqrt{9}$ $\quad$ $\sqrt{-1}\ \sqrt{4}$

$i \cdot 3$ $\quad$ $i \cdot 2$

$3i \quad \cdot \quad 2i$

$3i \cdot 2i = 6i^2$

$6 \cdot i^2 = 6 \cdot -1 = \boxed{6}$

6. Suppose a and b are positive real numbers. Determine whether the following quantities are equal or not equal.

A. $\sqrt{a} \cdot \sqrt{b}$ and $\sqrt{-a} \cdot \sqrt{-b}$

Not Equal
↳ see 5A & 5D above

B. $\sqrt{-a} \cdot \sqrt{b}$ and $\sqrt{a} \cdot \sqrt{-b}$

Equal
↳ see 5B & 5C above

Write each expression in complex form. Simplify as needed.

7. $\sqrt{-50}$

$\sqrt{-2}$ $\sqrt{25}$

$\sqrt{2} \cdot \sqrt{-1} \cdot 5$

$\sqrt{2} \cdot i \cdot 5$

$\boxed{5i\sqrt{2}}$

8. $\dfrac{-2 \pm \sqrt{-8}}{4}$

$\sqrt{-8}$

$\sqrt{-1}$ $\sqrt{8}$

$i \cdot 2\sqrt{2}$

$2i\sqrt{2}$

$\dfrac{-2 \pm 2i\sqrt{2}}{4} = \boxed{\dfrac{-1 \pm i\sqrt{2}}{2}}$

9. $\dfrac{3 \pm \sqrt{-4-5}}{6}$

$\sqrt{-4-5} = \sqrt{-9}$

$\sqrt{-1}$ $\sqrt{9}$

$i \cdot 3$

$3i$

$\dfrac{3 \pm 3i}{6} = \boxed{\dfrac{1+i}{2}}$

10. $\sqrt{-20} + \sqrt{40}$

$\sqrt{-1}$ $\sqrt{20}$ $\sqrt{4}$ $\sqrt{10}$

i $\sqrt{4}\sqrt{5}$ $2\sqrt{10}$

$2\sqrt{5}$

$2i\sqrt{5} + 2\sqrt{10}$

$\boxed{2i\sqrt{5} + 2\sqrt{10}}$

11. $\sqrt{27} + \sqrt{-27}$

$\sqrt{9} \cdot \sqrt{3}$ $\sqrt{9} \cdot \sqrt{3} \cdot \sqrt{-1}$

$3\sqrt{3}$ $3 \cdot \sqrt{3} \cdot i$

$3\sqrt{3} + 3i\sqrt{3}$

$\boxed{3\sqrt{3} + 3i\sqrt{3}}$

12. $\dfrac{4 \pm \sqrt{-28}}{8}$

$\sqrt{-28}$

$\sqrt{4}$ $\sqrt{7} \cdot \sqrt{-1}$

$2 \cdot \sqrt{7} \cdot i$

$2i\sqrt{7}$

$\dfrac{4 \pm 2i\sqrt{7}}{8} = \boxed{\dfrac{2 \pm i\sqrt{7}}{4}}$