NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

1. Determine the number and type of each solution for the following quadratic equations. b^2-4ac

A. $x^2 - 6x + 8 = 0$

b^2-4ac: $(-6)^2-4(1)(8)$

$36-32=4$

TWO REAL SOLUTIONS

B. $x^2 - 8x + 16 = 0$

b^2-4ac: $(-8)^2-4(1)(16)$

$64-64=0$

ONE REAL SOLUTION

C. $4x^2 + 1 = 0$

b^2-4ac $0^2-4(4)(1)$

$0-16=-16$

TWO complex solutions

2. Give a new example of a quadratic equation in standard form that has. . .

A. Exactly two distinct real solutions.

$f(x)= x^2-16$

B. Exactly one distinct real solution.

$f(x)= x^2+10x+25$

C. Exactly two complex (non-real) solutions.

$f(x)= x^2+9$

3. Suppose we have a quadratic equation $ax^2 + bx + c = 0$ so that $a + c = 0$. Does the quadratic equation have one solution or two distinct solutions? Are they real or complex? Explain how you know.

If $a+c=0$, then either $a=c=0$, $a>0$ and $c<0$, or $a<0$ and $c>0$.

- The definition of a quadratic polynomial requires that $\boldsymbol{a \neq 0}$, so either $\boldsymbol{a>0}$ and $\boldsymbol{c<0}$ or $\boldsymbol{a<0}$ and $\boldsymbol{c>0}$.
- In either case, $\boldsymbol{4ac<0}$. Because $\boldsymbol{b^2}$ is positive and $\boldsymbol{4ac}$ is negative, we know $\boldsymbol{b^2-4ac>0}$.
- Therefore, a quadratic equation $\boldsymbol{ax^2+bx+c=0}$ always has two distinct real solutions when $\boldsymbol{a+c=0}$.

4. Write a quadratic equation in standard form such that -5 is its only solution.

$(x+5)^2=0$

$x^2+10x+25=0$

5. Is it possible that the quadratic equation $ax^2 + bx + c = 0$ has a positive real solution if a, b, and c are all positive real numbers? NO

   A. What are the two solutions to the quadratic equation $ax^2 + bx + c = 0$?

   $$\frac{-b+\sqrt{b^2-4ac}}{2a} \qquad \frac{-b-\sqrt{b^2-4ac}}{2a}$$

   B. When will these solutions be positive?

If b is positive, the second will be negative
- If $-b+\sqrt{b^2-4ac} > 0$ then $\sqrt{b^2-4ac} > b$
- so $b^2-4ac > b^2$ and $-4ac > 0$

This means a or c must be negative.

*Therefore, if all 3 coefficients are positive, then there cannot be a positive solution to $ax^2+bx+c=0$

6. Is it possible that the quadratic equation $ax^2 + bx + c = 0$ has a positive real solution if a, b, and c are all negative real numbers? Explain your thinking.

No, if a, b, and c are all negative, then $-a$, $-b$, and $-c$ are all positive. The solutions of $ax^2+bx+c=0$ are the same as solutions to $-ax^2-bx-c=0$.

$-1(ax^2+bx+c)=0$

No positive real solutions since all positive coefficients

Solve.

7. $2x^2 + 8 = 0$

$2x^2+8=0$

$2x^2=-8$

$x^2=\sqrt{-4}$

$\pm\sqrt{4}\cdot\sqrt{-1}$

$2\cdot i$

$\boxed{\pm 2i}$

8. $x^2 + 5x + 12 = 0$

$$=\frac{-5\pm\sqrt{(5)^2-4(1)(12)}}{2(1)}=\frac{-5\pm\sqrt{25-48}}{2}$$

$$=\frac{-5\pm\sqrt{-23}}{2}=\boxed{\frac{-5\pm i\sqrt{23}}{2}}$$

9. $4x^2 - 2x + 2 = 0$

$$\frac{-(-2)\pm\sqrt{(-2)^2-4(4)(2)}}{2(4)}=\frac{2\pm\sqrt{4-32}}{8}$$

$$\frac{2\pm\sqrt{-28}}{8}=\frac{\cancel{2}^{1}\pm \cancel{2}^{1}i\sqrt{7}}{\cancel{8}_{4}}=\boxed{\frac{1\pm i\sqrt{7}}{4}}$$

10. $x^2 + 9 = 0$

$x^2=-9$

$x=\pm\sqrt{-9}$

$\boxed{x=\pm 3i}$

$\sqrt{-9}$

$\sqrt{9}$ $\sqrt{-1}$

$3\cdot i$

Homework Problem Set Sample Solutions

1. A. $x^2 - 6x + 8 = 0$
   $b^2 - 4ac =$
   $(-6)^2 - 4(1)(8) =$
   $36 - 32 = 4$
   $4 > 0$
   Two real solutions

   B. $x^2 - 8x + 16 = 0$
   $b^2 - 4ac$
   $(-8)^2 - 4(1)(16) =$
   $64 - 64 = 0$
   One real solution

   C. $4x^2 + 1 = 0$
   $b^2 - 4ac =$
   $0^2 - 4(4)(1) =$
   $0 - 16 =$
   $-16 < 0$
   Two complex solutions

2. Answers will vary.
   Have students check each other's equations. Ask how they are able to write an equation – What did they do to create their equations?

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2. Give a new example of a quadratic equation in standard form that has. . .

   A. Exactly two distinct real solutions.

   B. Exactly one distinct real solution.

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4. Write a quadratic equation in standard form such that -5 is its only solution.

3. If $a + c = 0$, then either $a = c = 0$, a > 0 and $c < 0$, or $a < 0$ and $c > 0$.
   - The definition of a quadratic polynomial requires that $\boldsymbol{a \neq 0}$, so either $\boldsymbol{a > 0}$ and $\boldsymbol{c < 0}$ or $\boldsymbol{a < 0}$ and $\boldsymbol{c > 0}$.
   - In either case, $\boldsymbol{4ac < 0}$. Because $\boldsymbol{b^2}$ is positive and $\boldsymbol{4ac}$ is negative, we know $\boldsymbol{b^2 - 4ac > 0}$.
   - Therefore, a quadratic equation $\boldsymbol{ax^2 + bx + c = 0}$ always has two distinct real solutions when $\boldsymbol{a + c = 0}$.

4. $\boldsymbol{(x+5)^2 = 0}$
   $\boldsymbol{x^2 + 10x + 25 = 0}$