

LESSON

5

Multiplying Binomials to Find the y -Intercept

LEARNING OBJECTIVES

- Today I am: learning two methods for multiplying polynomials.
- So that I can: rewrite quadratic functions in vertex form to standard form.
- I'll know I have it when I can: multiply $(3x^2 + 4x + 2)$ by $(2x + 3)$.

Opening Exercise

vertex form \rightarrow standard
 $y = a(x-h)^2 + k \rightarrow y = ax^2 + bx + c$
 \hookrightarrow y -inter.

1. Marshall had the quadratic function $y = (x - 2)^2 + 0$ and knew that he could easily find the vertex. What is the vertex for this quadratic function?

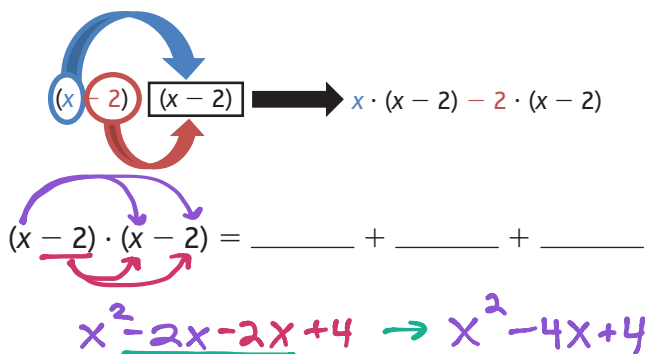
Sarah said that Marshall could find the y -intercept if he rewrote his equation into standard form, $y = ax^2 + bx + c$. Sarah got him started by rewriting his equation as shown below.

$$y = (x - 2)^2 + 0 \rightarrow \begin{array}{l} \text{vertex: } (2, 0) \\ \text{axis of sym: } x = 2 \\ \text{y-inter } (0, 4) \end{array}$$
$$y = (x - 2) \cdot (x - 2) + 0$$

Unfortunately, Marshall doesn't know how to multiply the binomials $(x - 2)$ by $(x - 2)$. In Module 2, Marshall learned to multiply -2 by $(x - 2)$.

$\frac{1}{2}(x-2)$
 $(x-2)(x-2)?$

Let's see how that idea can be used to tackle this new problem.



2. This distribution process continues by multiplying x by $(x - 2)$ and -2 by $(x - 2)$. Use the space above to continue the multiplication. Then combine like terms to get a 3-term (trinomial) expression.

$y = (x-2)^2 + 0$
 $y = x^2 - 4x + 4 \rightarrow y\text{-intercept}$
 How?

3. What is the y -intercept for Marshall's quadratic function?

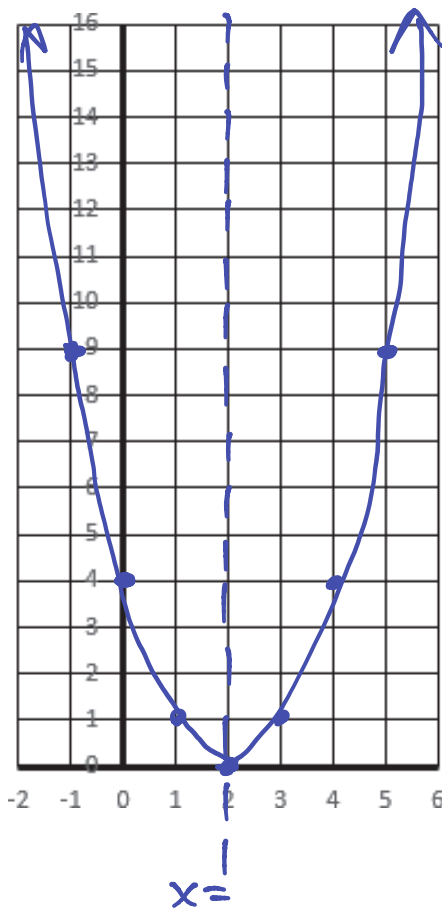
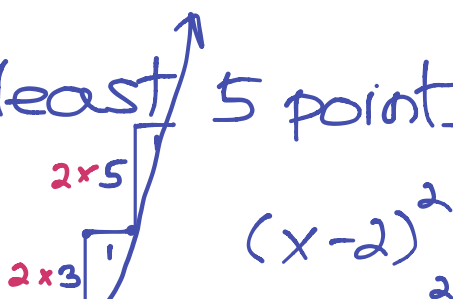
$y\text{-intercept, let } x=0$
 $y = 0^2 - 4(0) + 4$
 $y = 4 \quad (0, 4)$

x	y	
1	1	$(1-2)^2 + 0$
5	9	$(5-2)^2 + 0$

4. Use the vertex and y -intercept to sketch a graph of Marshall's parabola. What could you do to get a more accurate graph?

vertex: $(2, 0)$
 axis of sym: $x=2$
 y -inter: $(0, 4)$

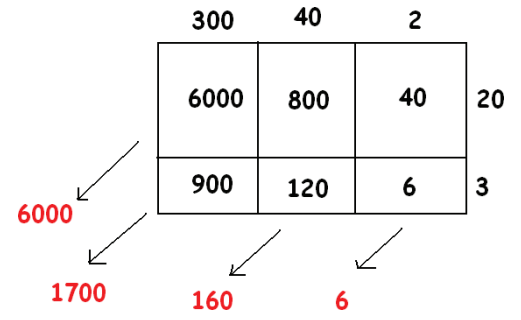
(At least 5 points)



Exploratory Challenge—The Area Model

5. Gisella computed 342×23 as shown on the right.

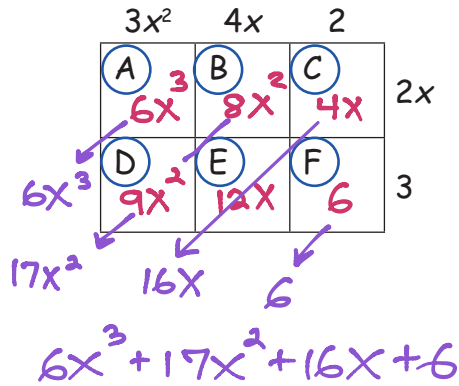
Explain what she is doing. What is her final answer?



6. Bobby said that Gisella’s method could be used in algebra to multiply other polynomials. He wrote the following:

$(3x^2 + 4x + 2)(2x + 3) = \text{????}$

$\underbrace{\hspace{2cm}}_3$ $\underbrace{\hspace{2cm}}_2$



Determine what goes in each box by simplifying the expressions below.

- A. $3x^2 \cdot 2x = \underline{\hspace{2cm}}$ B. $4x \cdot 2x = \underline{\hspace{2cm}}$ C. $2 \cdot 2x = \underline{\hspace{2cm}}$
 D. $3x^2 \cdot 3 = \underline{\hspace{2cm}}$ E. $4x \cdot 3 = \underline{\hspace{2cm}}$ F. $2 \cdot 3 = \underline{\hspace{2cm}}$

7. How can you use the arrows like Gisella had in her problem to help you simplify the expression?

$6x^3 + 17x^2 + 16x + 6$

8. $(3x^2 + 4x + 2)(2x + 3) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$

Both models used in Exercises 5 and 6 were area models. You will use these models again in Algebra 2 when you multiply complicated polynomials and divide polynomials.

Sometimes the quadratic function is given in factored form, such as $y = (x - 1)(x + 2)$. We'll use the area model to change the equation into standard form.

9. To multiply $(x - 1)$ and $(x + 2)$, we'll set up a table to keep track of the terms.

$(x+2) (x-1)$

	x	-1	
x^2	$-x$	x	
$2x$	-2	2	

Where does the x^2 term come from?
 $x \cdot x$

What should go in this box?

One more box to go!

x^2

x

-2

Use the arrows to help you combine like terms. We know that

$(x - 1) \cdot (x + 2) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$.

$y = x^2 + x - 2$

10. What is the y-intercept for the equation $y = (x - 1)(x + 2)$?

$y = x^2 + x - 2$ -2 $\rightarrow (0, -2)$

11. What is another way to find the y-intercept?

let $x = 0$
 $y = (0)^2 + 0 - 2$
 $y = -2$

In later lessons, we'll learn about standard form and factored form and what information each gives us about the graph of the quadratic equation. For now, we'll focus on multiplying various expressions.

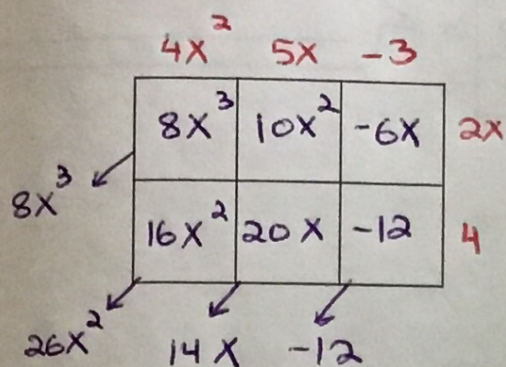
Lesson 5 Multiplying Binomials to Find the y-Intercept

12. For the next two exercises, you and your partner will switch off using the area model and the distributive property. When you are done, check your answers with your partner's.

Partner A's Work

- A. Use an area model to find the simplified expression for

$$(4x^2 + 5x - 3) \cdot (2x + 4)$$



$$(4x^2 + 5x - 3) \cdot (2x + 4) = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$= 8x^3 + 26x^2 + 14x - 12$$

- B. Use the distributive property to find the simplified expression for

$$(x - 3) \cdot (x^2 + 6x - 9)$$

$$x(x^2 + 6x - 9) = x^3 + 6x^2 - 9x$$

$$-3(x^2 + 6x - 9) = -3x^2 - 18x + 27$$

$$x^3 + 3x^2 - 27x + 27$$

$$(x - 3) \cdot (x^2 + 6x - 9) = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$x^3 + 3x^2 - 27x + 27$$

Partner B's Work

- A. Use the distributive property to find the simplified expression for

$$(4x^2 + 5x - 3) \cdot (2x + 4)$$

$$2x(4x^2 + 5x - 3) = 8x^3 + 10x^2 - 6x$$

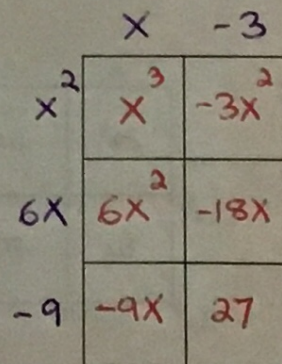
$$4(4x^2 + 5x - 3) = 16x^2 + 20x - 12$$

$$8x^3 + 26x^2 + 14x - 12$$

$$(4x^2 + 5x - 3) \cdot (2x + 4) = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

- B. Use an area model to find the simplified expression for

$$(x - 3) \cdot (x^2 + 6x - 9)$$



$$(x - 3) \cdot (x^2 + 6x - 9) = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$x^3 + 3x^2 - 27x + 27$$

13. Ian started to multiply the polynomials, $(x - 1)$ and $(x^3 + 6x^2 - 5)$, using an area model. Answer the questions to help Ian finish the problem.

Where does the x^4 come from?

 Fill in the other boxes.

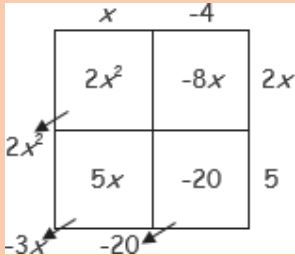
This space will have the sum of the two like terms with an x^2 . What is that sum? _____
 Determine the other sums of like terms.

Why did Ian put a $0x$ in the problem?
 $0x$ is a place holder for the missing term.

Using this area model, we can see that $(x - 1) \cdot (x^3 + 6x^2 - 5) = \underline{\hspace{1cm}} + 5x^3 + \underline{\hspace{1cm}} - 5x + \underline{\hspace{1cm}}$.

Lesson Summary

Two different methods for multiplying polynomials are the area model and using the distributive property. An example of each is shown below:



$$\begin{aligned} (x - 4)(2x + 5) &= \\ x(2x + 5) - 4(2x + 5) &= \\ 2x^2 + 5x - 8x - 20 &= \\ 2x^2 - 3x - 20 & \end{aligned}$$

14. Which method do you like best? Why?

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

Use an area model *box method* to compute the following products:

1. $(4x + 2)(2x + 3)$

2. $(10x + 1)(x + 1)$

Hint: For Problems 3 and 4, use ~~0-terms~~ as shown in Exercise 13.

3. $(3x^2 + 2)(2x + 3)$

4. $(2x^2 + 10x)(x^2 + 1)$

5. $(3x^2 + 4x + 2)(2x + 3)$

6. $(2x^2 + 10x + 1)(x^2 + x + 1)$

7. Multiply the polynomials using the distributive property: $(3x^2 + x - 1)(x^4 - 2x + 1)$.

$$(3x^2 + x - 1)(x^4 - 2x + 1)$$

$$3x^2(x^4 - 2x + 1) = 3x^6 - 6x^3 + 3x^2$$

$$x(x^4 - 2x + 1) = x^5 - 2x^2 + x$$

$$-1(x^4 - 2x + 1) = -x^4 + 2x - 1$$

$$3x^6 + x^5 - x^4 - 6x^3 + x^2 + 3x - 1$$

8. Sammy wrote a polynomial using only one variable, x , of degree 3. Myisha wrote a polynomial in the same variable of degree 5. What can you say about the degree of the product of Sammy's and Myisha's polynomials?

$$(x^3 + \dots) \cdot (x^5 + \dots)$$

$$(x^8 + \dots)$$

Lesson 5 Multiplying Binomials to Find the y-Intercept

Use either method to write each of the following expressions as the sum of monomials.

9. $3a(4 + a)$
 $12a + 3a^2$

10. $x(x + 2) + 1$

11. $(x - 4)(x + 5)$

12. $(2z - 1)(3z^2 + 1)$

13. $(10w - 1)(10w + 1)$

14. $(-5w - 3)w^2$

15. $(x^2 - x + 1)(x - 1)$

16. $(x + y)(y + z)(z + x)$

$$(xy + xz + y^2 + yz)(z + x)$$

$$z(xy + xz + y^2 + yz)$$

$$x(xy + xz + y^2 + yz)$$

$$\rightarrow \boxed{xyz} + xz^2 + y^2z + yz^2$$

$$\rightarrow x^2y + x^2z + xy^2 + \boxed{xyz}$$

$$\boxed{2xyz + xz^2 + y^2z + yz^2 + x^2y}$$

$$+x^2z + xy^2$$

17. $(t - 1)(t + 1)(t^2 + 1)$

18. $(w + 1)(w^4 - w^3 + w^2 - w + 1)$

19. $z(2z + 1)(3z - 2)$

20. $3xz(9xy + z) - 2yz(x + y - z)$

Be careful here!

You'll need to multiply each term separately. Then combine like terms.

Lesson 5 Multiplying Binomials to Find the y-Intercept

21. Use the distributive property (and your wits!) to write each of the following expressions as a sum of monomials. If the resulting polynomial is in one variable, write the polynomial in standard form.

- A. $(a + b)^2$ B. $(a + 1)^2$ C. $(3 + b)^2$ D. $(3 + 1)^2$

E. What do you notice about all of these problems? Is there a pattern?

22. Andrew started to multiply the polynomials, $(x - 1)$ and $(x^3 + 6x^2 - 5)$, using the distributive property. Examine Andrew's work and then complete the problem.

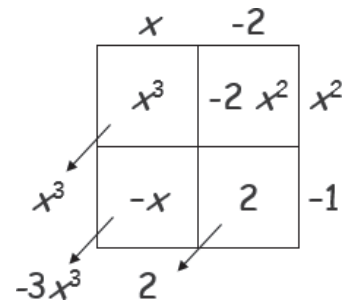
$$(x - 1) \cdot (x^3 + 6x^2 - 5) =$$

$$x \cdot (x^3 + 6x^2 - 5) - 1(x^3 + 6x^2 - 5) =$$

$$\underline{\hspace{10em}} = \underline{\hspace{2em}} + \underline{\hspace{2em}} + \underline{\hspace{2em}} + \underline{\hspace{2em}} + \underline{\hspace{2em}}$$

23. Leela is convinced that $(a + b)^2 = a^2 + b^2$. Use an area model to explain to her why she is wrong.

24. Sara started to use the area model to multiply $(x - 2)$ by $(x^2 - 1)$. Explain where Sara went wrong in her area model. What could she have done to prevent this mistake?



Challenge Problems

25. $(x + y + z)^2$

26. $(x + 1 + z)^2$

27. The expression $10x^2 + 6x^3$ is the result of applying the distributive property to the expression $2x^2(5 + 3x)$. It is also the result of applying the distributive property to $2(5x^2 + 3x^3)$ or to $x(10x + 6x^2)$, for example, or even to $1 \cdot (10x^2 + 6x^3)$. For (A) to (E) below, write down an expression such that if you applied the distributive property to your expression, it would give the result presented. Give interesting answers!

Example: $10x^2 + 6x^3$ can be written as:

$$2x^2(5 + 3x)$$

A. $6a + 14a^2$ can be written as:

B. $2x^4 + 2x^5 + 2x^{10}$ can be written as:

C. $6z^2 - 15z$ can be written as:

D. $42w^3 - 14w + 77w^5$ can be written as:

E. $z^2(a + b) + z^3(a + b)$ can be written as:

