LESSON 6

Focus on the Standard Form, $f(x) = ax^2 + bx + c$

LEARNING OBJECTIVES

- Today I am: exploring a baseball problem and graphing the path of the ball.
- So that I can: rewrite a quadratic function in vertex form to standard form.
- I'll know I have it when I can: determine what graphing information each form gives.

Opening Exploration—Focus on the Equation

1. A high school baseball player throws a ball straight up into the air for his math class. The math class was able to determine that the relationship between the height of the ball and the time since it was thrown could be modeled by the function

y-inter: (0,6)

concave down

$$h(t) = -16t^2 + 96t + 6,$$

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where t represents the time (in seconds) since the ball was thrown, and h represents the height (in feet) of the ball above the ground.

A. What is the domain (input) of this function?

[0, 6.0]

B. What is the range (output) of this function?

[0, 150]

C. What is the time (t) when the ball is thrown?

0 second

D. What is the starting height of the ball?

6 feet (y-inter)

E. What is the vertex of this parabola? How did you determine the vertex?

(3, 150) the maximum

F. What is the maximum height that the ball reaches while in the air?

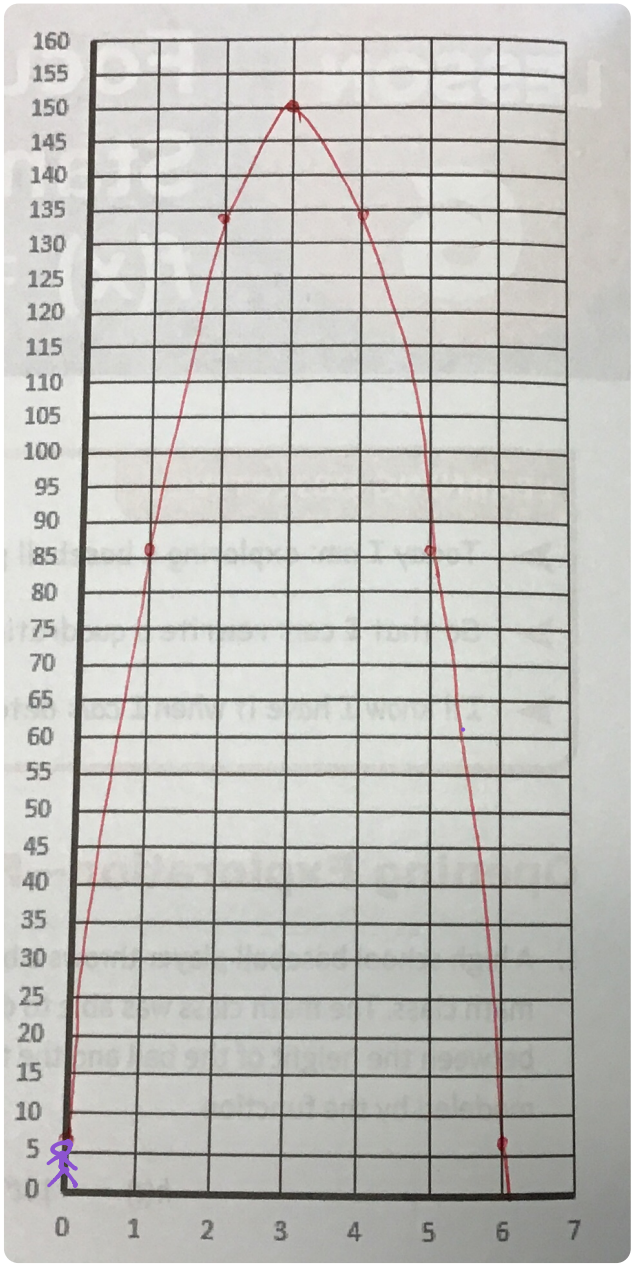
150 ft

G. How long will the ball take to reach its maximum height?

3 seconds

2. It would be difficult to tell from the equation how many seconds it takes the ball to hit the ground.

   A. Graph the equation $h(t) = -16t^2 + 96t + 6$, where t represents the time (in seconds) since the ball was thrown, and h represents the height (in feet) of the ball above the ground in the grid at the right and label the axes. Then make an estimate. A table is given below to help you graph this function.



t	$h(t) = -16t^2 + 96t + 6$
0	$h(0) = -16(0)^2 + 96(0) + 6 = 6$
1	$h(1) = -16(1)^2 + 96(1) + 6 = 86$
2	$h(2) = -16(2)^2 + 96(2) + 6 = 134$
3	150
4	134
5	86
6	6
7	−106

Approximately, when does the ball hit the ground?

6.062 seconds

B. What does the domain represent?

The time the ball was in the air.

C. What does the range of this function represent?

the height of the ball.

Discussion

3. A. The equation in the Opening Exploration was in standard form. What feature(s) of a quadratic function are *visible* when it is presented in the standard form, $f(x) = ax^2 + bx + c$?

- y-intercept → (0, c)
- concavity.

B. What feature(s) of a quadratic function are *visible* when it is written in vertex form, $f(x) = a(x - h)^2 + k$?

- vertex (h, k)
- stretch / shrink (vertically)
- concavity.
- axis of symmetry: $x = h$

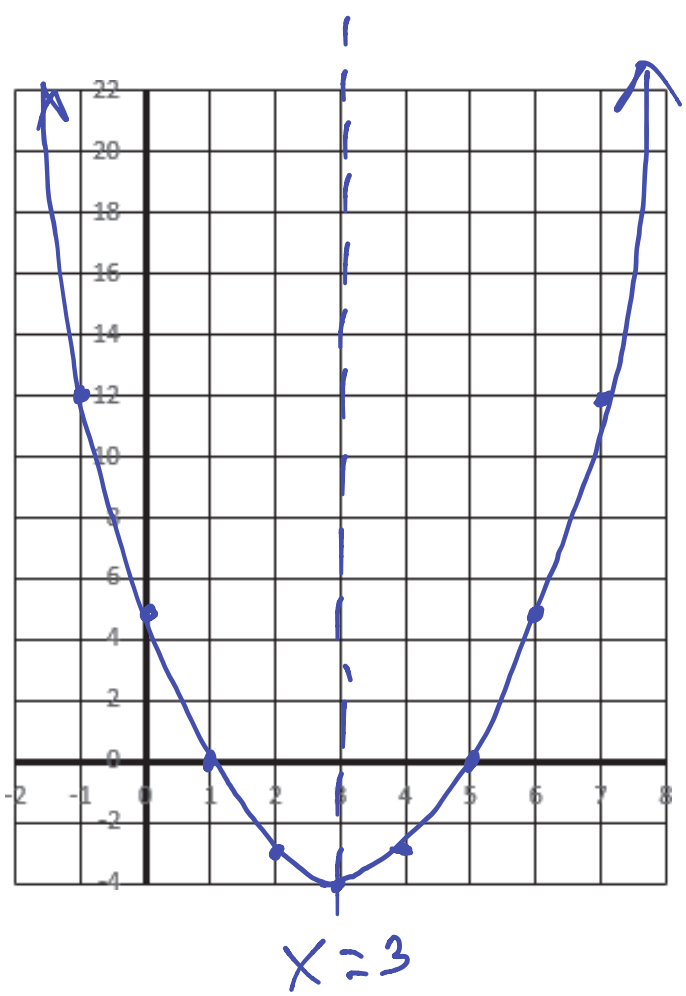
4. A general strategy for graphing a quadratic function from the standard form is:

5. Graph the function $n(x) = x^2 - 6x + 5$, and identify the key features.

y-inter: (0, 5)

concave up

x	n(x)
-1	$(-1)^2 - 6(-1) + 5 = 12$
1	$(1)^2 - 6(1) + 5 = 0$
2	$(2)^2 - 6(2) + 5 = -3$
3	$(3)^2 - 6(3) + 5 = -4$
4	$(4)^2 - 6(4) + 5 = -3$



Practice: Vertex Form to Standard Form

For each exercise below, rewrite the vertex form into standard form and then identify all the important features of each quadratic.

6. Vertex form: $y = (x + 6)^2 - 4$

$y = (x+6)(x+6) - 4$

$y = x^2 + 6x + 6x + 36 - 4$

$y = x^2 + 12x + 32$

Standard form: $y = x^2 + 12x + 32$

Key Features

Vertex: $(-6, -4)$

y-intercept: $(0, 32)$

Does the graph open up or open down?

concave up

Axis of symmetry: $x = -6$

7. Vertex form: $y = (x - 1)^2 - 7$

Key Features

Vertex: _____

y-intercept: _____

Does the graph open up or open down?

_____

Standard form: _____

Axis of symmetry: _____

8. Vertex form: $y = -(x - 3)^2 + 2$

$y = -(x-3)(x-3) + 2$

$= -(x^2 - 3x - 3x + 9) + 2$

$= -(x^2 - 6x + 9) + 2$

$= -x^2 + 6x - 9 + 2$

Standard form: $y = -x^2 + 6x - 7$

Key Features

Vertex: $(3, 2)$

y-intercept: $(0, -7)$

Does the graph open up or open down?

concave down

Axis of symmetry: $x = 3$

Lesson Summary

The standard form of a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$. From the standard form you can easily see that the *y*-intercept is at *c*.

A general strategy to graphing a quadratic function from the standard form:

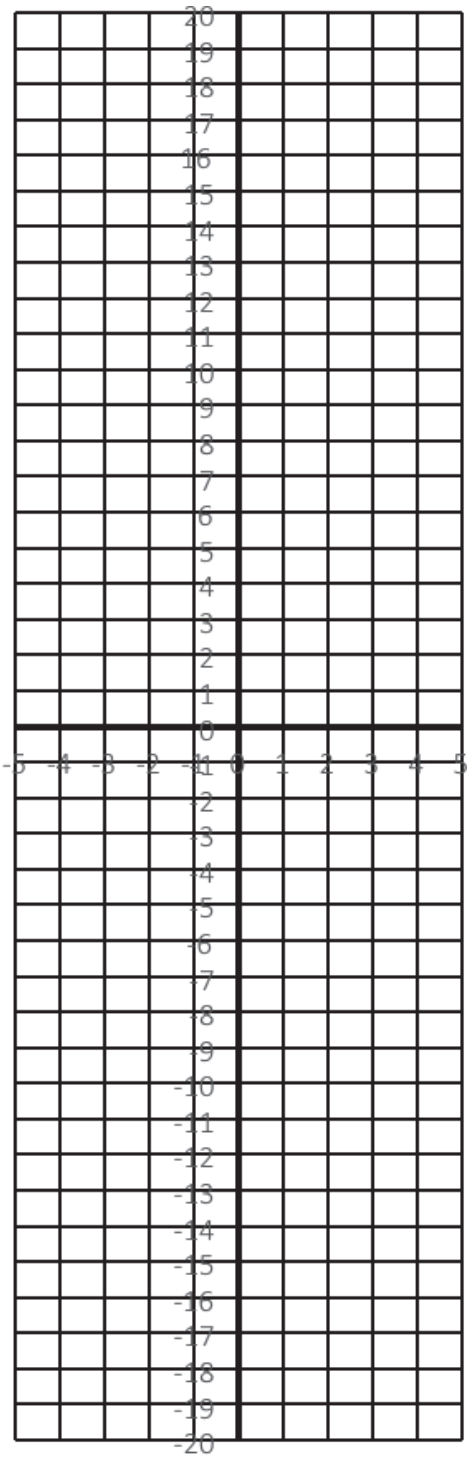
- Look for hints in the function's equation for general shape, direction, and *y*-intercept.
- Use a T-chart to find more points on the graph.
- Remember that a parabola is symmetric about the vertex. This can help you identify other points you may need.
- Plot the points that you know (at least three are required for a unique quadratic function), sketch the graph of the curve that connects them, and identify the key features of the graph.

NAME: _____ PERIOD: _____ DATE: _____

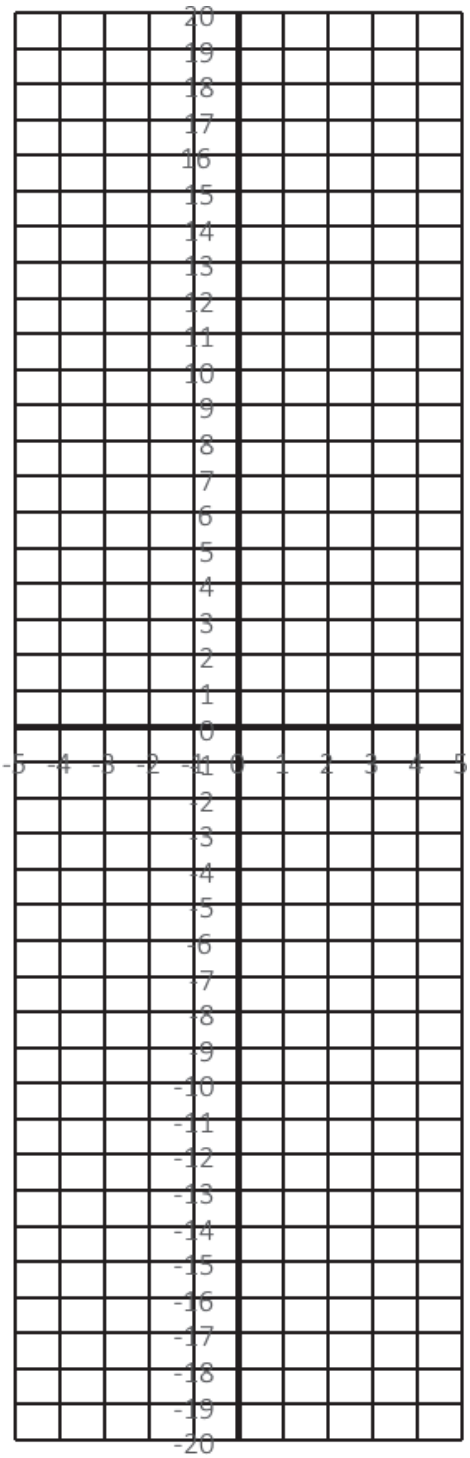
Homework Problem Set

1. Graph each quadratic equation given both the standard and vertex forms below.

   A. $f(x) = x^2 - 2x - 15$ or
   $f(x) = (x - 1)^2 - 16$



   B. $f(x) = -x^2 + 2x + 15$ or
   $f(x) = -(x - 1)^2 + 16$



2. The equation in Part B of Problem 1 is the product of -1 and the equation in Part A. What effect did multiplying the equation by -1 have on the graph?

3. Paige wants to start a summer lawn-mowing business. She comes up with the following profit function that relates the total profit to the rate she charges for a lawn-mowing job:

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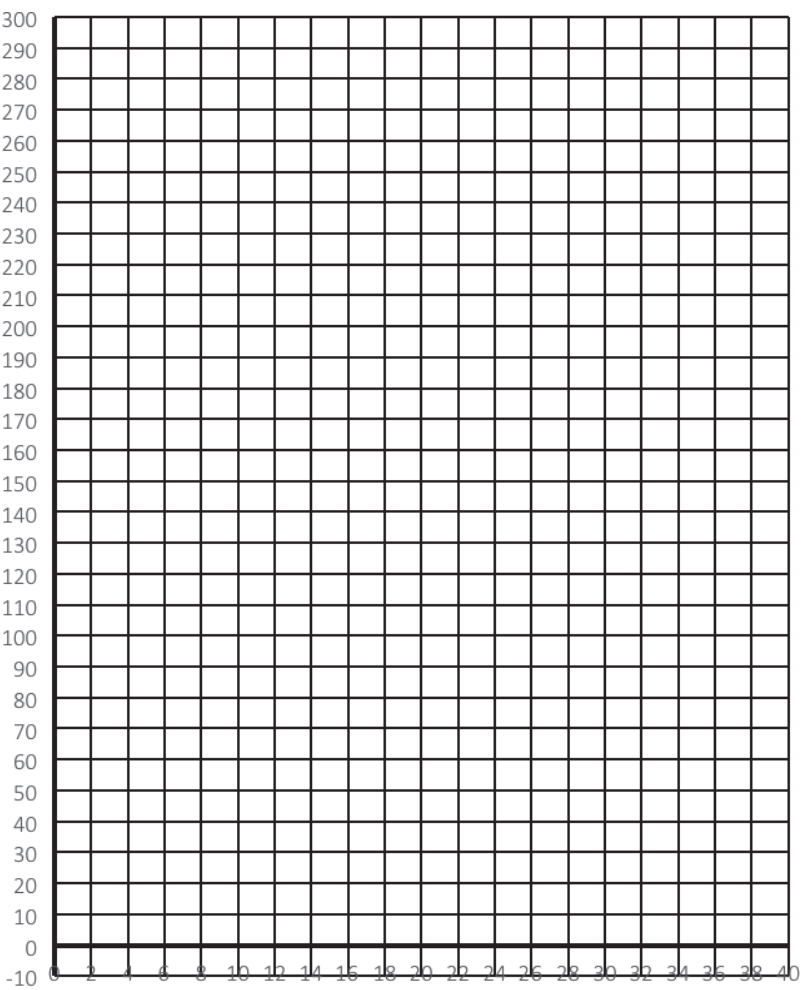
$$P(x) = -x^2 + 40x - 100$$

Both profit and her rate are measured in dollars.

A. Graph the function to help you answer the following questions. A table is given below to help you graph the function.

x	$P(x) = -x^2 + 40x - 100$
0	
4	
8	
12	
16	
20	
24	
28	
32	
36	
40	



B. According to the function, what is her initial cost (e.g., maintaining the mower, buying gas, advertising)? Explain your answer in the context of this problem.

C. Between what two prices does she have to charge to make a profit?

D. If she wants to make $275 profit this summer, is this the right business choice? Explain.

4. A student throws a bag of chips to her friend. Unfortunately, her friend does not catch the chips, and the bag hits the ground. The distance from the ground (height) for the bag of chips is modeled by the function $h(t) = -16(t - 1)^2 + 20$, where h is the height (distance from the ground in feet) of the chips, and t is the number of seconds the chips are in the air.

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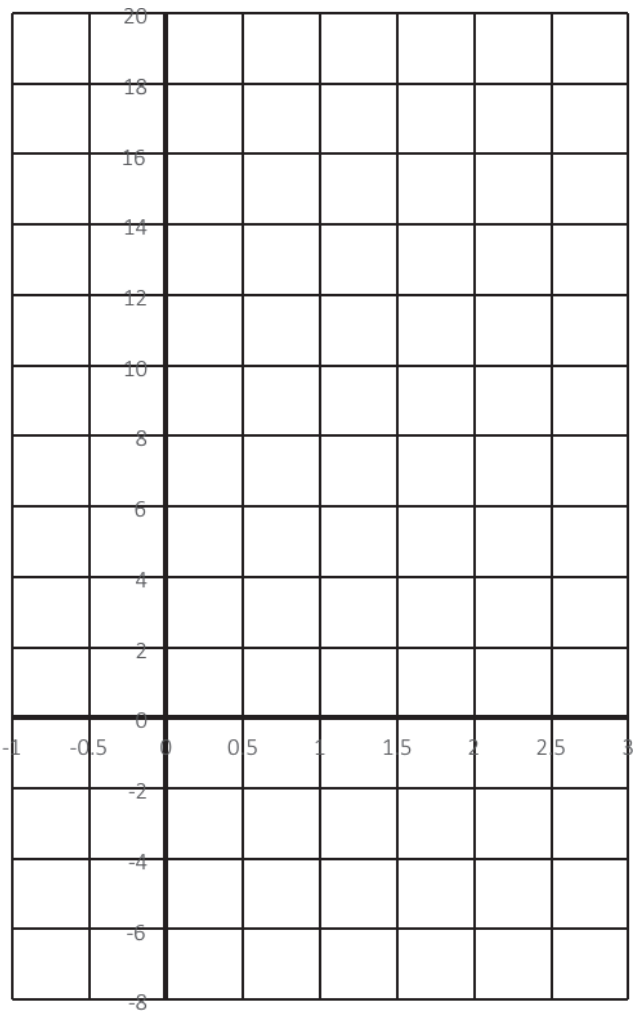
   A. Graph $h(t) = -16(t - 1)^2 + 20$.

   B. From what height are the chips being thrown? Explain how you know.



   C. What is the maximum height the bag of chips reaches while airborne? Explain how you know.

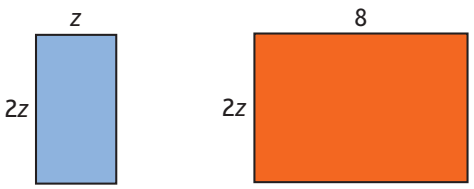
   D. About how many seconds after the bag was thrown did it hit the ground?

   E. What is the average rate of change of height for the interval from 0 to $\frac{1}{2}$ second? What does that number represent in terms of the context?

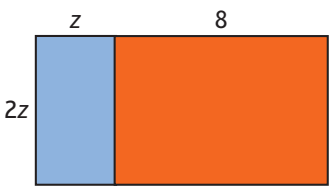
   F. Based on your answer to Part E, what is the average rate of change for the interval from 1.5 to 2 sec.?

Spiral REVIEW—Writing Expressions for Area

5. Write expressions for the areas of the two rectangles in the figures given below.



6. Write an expression for the area of this rectangle:



7. Katy says that the two expressions in Problems 5 and 6 must be the same. Support or oppose Katy's statement with evidence.