LESSON 7

Further Explorations of Vertex Form, $y = a(x - h)^2 + k$

LEARNING OBJECTIVES

- Today I am: matching the key features of quadratic functions using game cards.
- So that I can: look for patterns in the axis of symmetry.
- I'll know I have it when I can: find the axis of symmetry in two ways.

Opening Exercise—Warm Up

1. Without graphing, state the vertex for each of the following quadratic equations.

   A. $y = (x - 5)^2 + 3$

   vertex: (5, 3)

   B. $y = x^2 - 2.5$

   $(x-0)^2-2.5$

   vertex: (0, -2.5)

   C. $y = (x + 4)^2$ + 0

   vertex: (-4, 0)

$y = a(x-h)^2 + k$

2. Write a quadratic equation whose graph will have the given vertex.

A. (1.9, −4)

$y = (x-1.9)^2 - 4$

assume that $a = 1$

B. (0, 100)

$y = x^2 + 100$

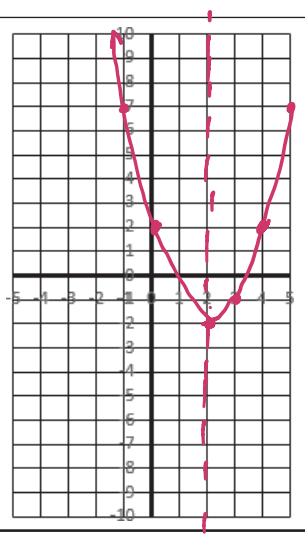
C. $\left(-2, \frac{3}{2}\right)$

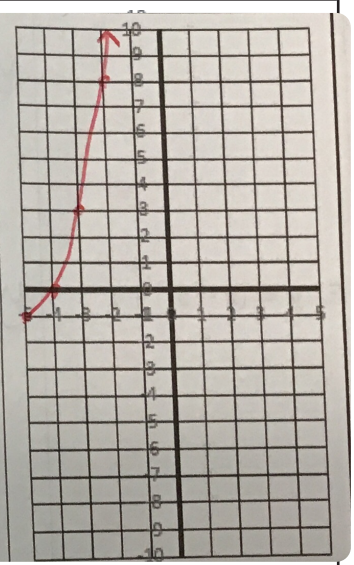
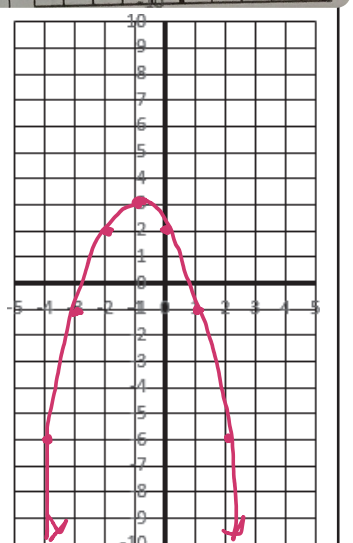
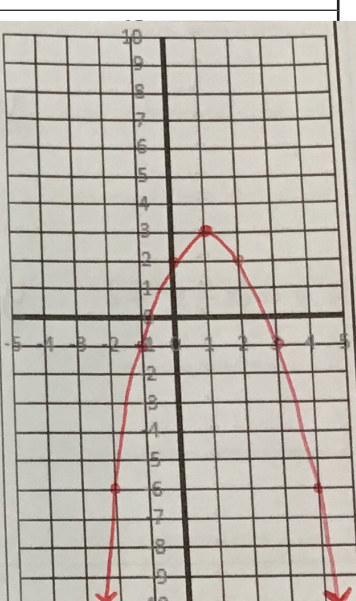
$y = (x+2)^2 + \frac{3}{2}$

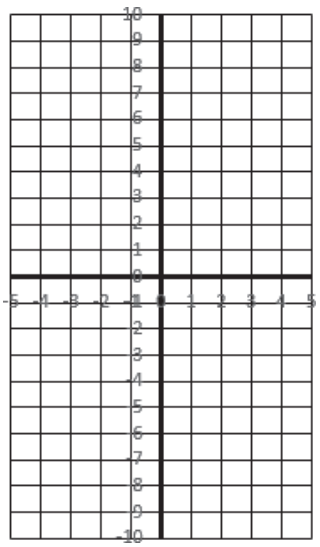
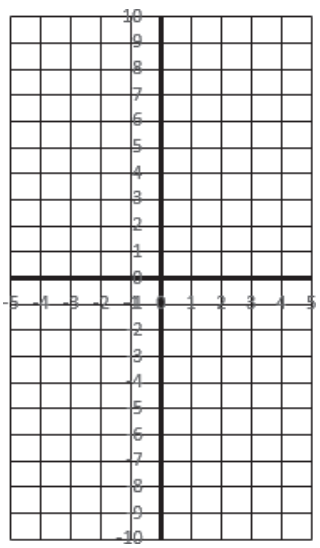
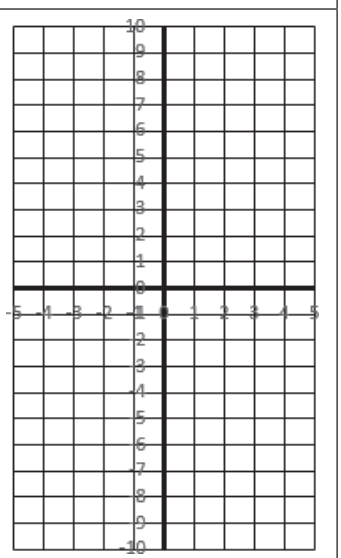
Exploratory Challenge

Your group will need: *Quadratic Matching* game cards

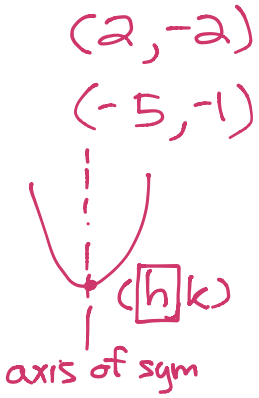
3. With your group, match four different aspects of a given quadratic function. There are cards for the vertex, equation in standard form, *y*-intercept and graph. Record your matches in the table below. Your graph should be a rough sketch, but put in at least 3 key points.

Equation in Vertex Form	Equation in Standard Form	Vertex	*y*-intercept	Graph
A. $y = (x-2)^2 - 2$ $y = (x-2)(x-2) - 2$ $y = x^2 - 4x + 4 - 2$ $y = x^2 - 4x + 2$	$y = x^2 - 4x + 2$	$(2, -2)$ $x = 2$	$(0, 2)$	

Equation in Vertex Form	Equation in Standard Form	Vertex	*y*-intercept	Graph
B. $y = (x + 5)^2 - 1$	$y = (x+5)(x+5)-1$ $y = x^2 + 10x + 24$	$(-5, -1)$	24	
C. $y = -(x + 1)^2 + 3$ $y = -(x+1)(x+1)+3$ $y = -(x^2+2x+1)+3$ $y = -x^2-2x-1+3$ $y = -x^2-2x+2$	$y = -x^2 - 2x + 2$	$(-1, 3)$ $x = -1$	$(0, 2)$	
reflect x-axis D. $y = -(x - 1)^2 + 3$	$y = -(x-1)(x-1)+3$ $y = -x^2 + 2x + 2$	$(1, 3)$	2	

Equation in Vertex Form	Equation in Standard Form	Vertex	*y*-intercept	Graph
E. $y = (x - 5)^2 + 1$				
F. $y = -(x + 2)^2 + 2$				
G. $y = (x + 1)^2 + 2$				

4. For each equation from Exercise 3, determine the axis of symmetry. Then draw the axis of symmetry on your graphs in Exercise 3.



Equation in Vertex Form	Axis of Symmetry
A. $y = (x - 2)^2 - 2$	$x = 2$
B. $y = (x + 5)^2 - 1$	$x = -5$
C. $y = -(x + 1)^2 + 3$	$x = -1$
D. $y = -(x - 1)^2 + 3$	$x = 1$
E. $y = (x - 5)^2 + 1$ (5,1)	$x = 5$
F. $y = -(x + 2)^2 + 2$ (-2,2)	$x = -2$
G. $y = (x + 1)^2 + 2$ (-1,2)	$x = -1$

5. Scott says that his tutor gave him the equation, $x = -\frac{b}{2a}$, to find the axis of symmetry of a quadratic function. The a and b are found when the equation is in standard form $f(x) = ax^2 + bx + c$. Use the standard forms of the equations from Exercise 3 to verify the axis of symmetry. (These are in no particular order.)

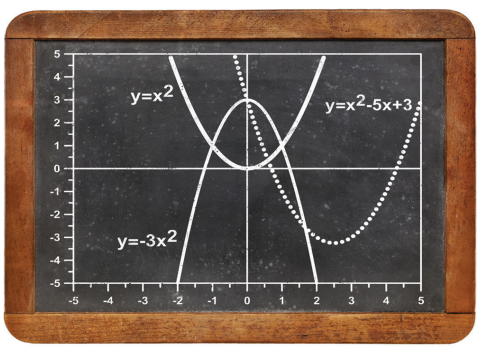
Equation in Standard Form	Axis of Symmetry Using $x = -\frac{b}{2a}$
A. $y = -x^2 + 2x + 2$ $a=-1$ $b=2$	$x = -\frac{2}{2(-1)} = \frac{-2}{-2} = 1 \rightarrow x = 1$ (1, ?)
B. $y = x^2 + 10x + 24$ $a=1$ $b=10$	$x = \frac{-10}{2(1)} = \frac{-10}{2} = -5 \rightarrow x = -5$
C. $y = -x^2 - 4x - 2$ $a=-1$ $b=-4$	$x = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2 \rightarrow x = -2$
D. $y = x^2 - 4x + 2$	
E. $y = x^2 + 2x + 3$	
F. $y = -x^2 - 2x + 2$	
G. $y = x^2 - 10x + 26$	

$h = \frac{-b}{2a}$ $\quad y = ax^2 + bx + c$

6. In the image below, two of the equations have an axis of symmetry at $x = 0$.

   A. What part of the equation tells you that any equation of the form $y = ax^2$ will have an axis of symmetry of $x = 0$?

   B. What is the axis of symmetry for the third quadratic function?



Lesson Summary

The standard form of a quadratic equation is $f(x) = ax^2 + bx + c$.

The vertex form of a quadratic equation is $f(x) = a(x - h)^2 + k$.

Both forms give specific information about the parabola formed.

You can use the equation $x = -\frac{b}{2a}$ with the standard form to find the axis of symmetry.

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. Find the vertex of the graphs of the following quadratic equations.

   A. $y = 2(x - 5)^2 + 3.5$

   B. $y = -(x + 1)^2 - 8$

For each problem below, identify which equations satisfy the given conditions. In some cases, there may only be one equation that works, while others have multiple equations that fulfill the requirements.

2. Vertex: $(3, -2)$

   ☐ $f(x) = 3x^2 + 2$

   ☐ $f(x) = (x - 3)^2 + 2$

   ☐ $f(x) = -(x - 3)^2 + 2$

   ☐ $f(x) = (x - 3)^2 - 2$

   ☐ $f(x) = -(x - 3)^2 - 2$

3. Vertex: $(1, 4)$; y-intercept: 5

   ☐ $f(x) = (x - 1)^2 + 4$

   ☐ $f(x) = x^2 - 2x + 5$

   ☐ $f(x) = -x^2 + 2x + 5$

   ☐ $f(x) = 2x^2 - 4x + 5$

   ☐ $f(x) = -(x - 1)^2 + 4$

4. y-intercept: 3

   ☐ $f(x) = x^2 + 3$

   ☐ $f(x) = x^2 - 2x + 3$

   ☐ $f(x) = -(x - 1)^2 + 4$

   ☐ $f(x) = -2(x + 1)^2 + 5$

5. Prove your results from Problem 2. (The equations are given at the right for your convenience.)

2. Vertex: $(3, -2)$

   ☐ $f(x) = 3x^2 + 2$

   ☐ $f(x) = (x - 3)^2 + 2$

   ☐ $f(x) = -(x - 3)^2 + 2$

   ☐ $f(x) = (x - 3)^2 - 2$

   ☐ $f(x) = -(x - 3)^2 - 2$

Determine the axis of symmetry for each quadratic equation below.

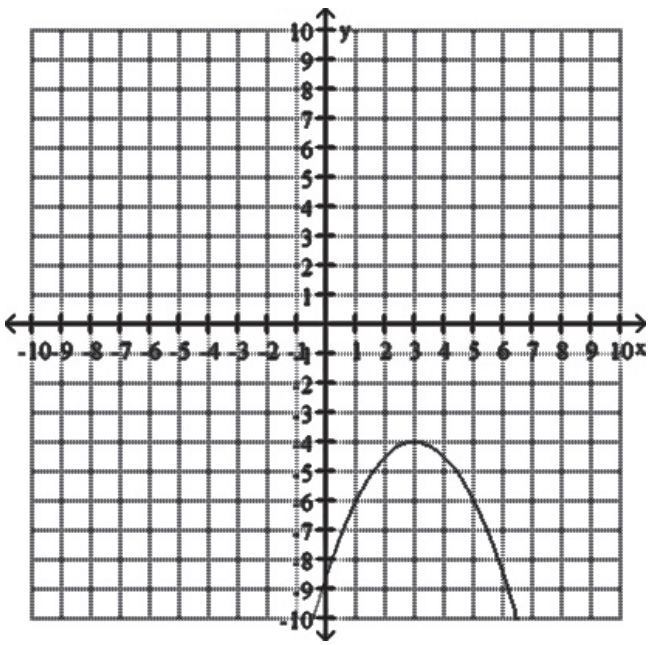
6. $f(x) = 3x^2 + 7$	7. $g(x) = x^2 - 2x + 3$	8. $h(x) = 2(x - 1)^2 + 2$
9. $y = -(x - 4)^2 + 8$	10. $m(x) = 3(x - 1)^2$	11. $y = \frac{1}{2}x^2 - 4x + 5$
12. $f(x) = \frac{1}{4}(x - 4)^2 - 2$	13. $g(x) = \frac{3}{2}x^2 + 6x - 4$	14. $y = (3x - 1)^2 + 3$

For each graph below, state the vertex, axis of symmetry and write the equation of each function.

15. Vertex: _____

Axis of Symmetry: _____

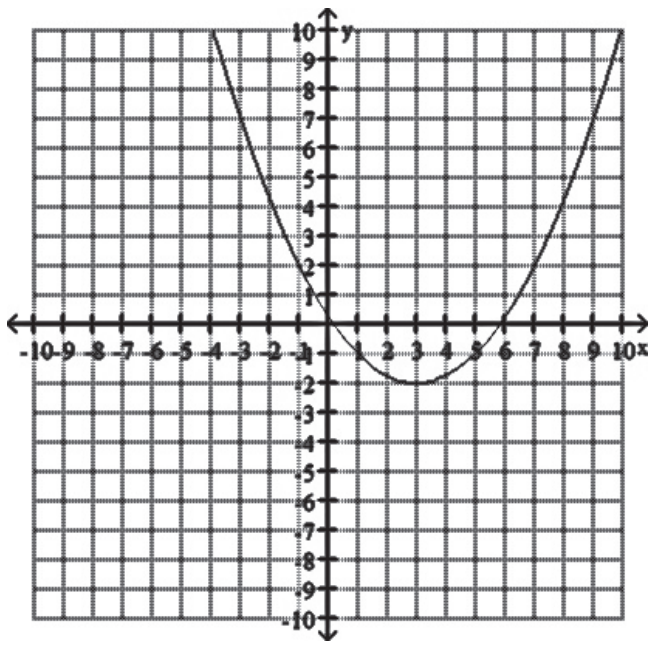
$y = -\frac{1}{2}(x - \_\_\_\_)^2 - \_\_\_\_$



16. Vertex: _____

Axis of Symmetry: _____

$y = \frac{1}{4}(x - \_\_\_\_)^2 - \_\_\_\_$



17. Write an equation of a quadratic function that has an axis of symmetry of $x = 0$.