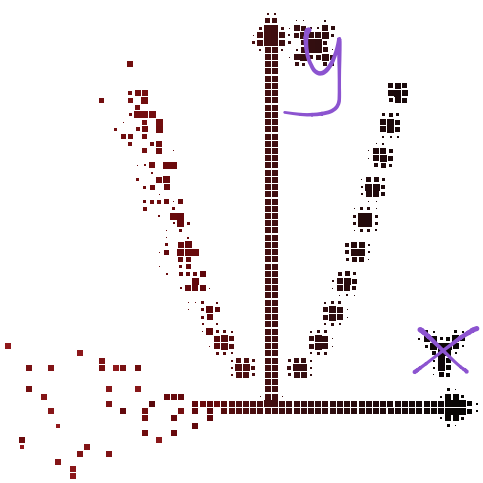
LESSON 8

Quadratic Transformations—Taking It to the Next Level

LEARNING OBJECTIVES

- Today I am: sorting graphs of parabolas.
- So that I can: identify what each type of transformation does to a quadratic function.
- I'll know I have it when I can: clarify the transformation ideas in abstract form as they relate to quadratics.

Back in Unit 7, you looked at transformations with a variety of functions, including quadratic functions. Throughout this unit, we've examined transformations with quadratic functions, but now it is time to solidify our thinking and take it to the next level.



Opening Activity

You will need: *Graphs of Quadratic Functions* cards, glue stick or tape, scissors

1. Cut out and sort out the cards and then choose the ones you think best represent the transformation given. There are multiple correct matches and some graphs shouldn't be used at all, so don't be confused by having left over cards. Then write the equation in vertex form for each example you chose. The transformed function is a dashed curve.

Equation in Abstract Form	Transformation	Example Graph	Equation of Example Graph
$y = f(x) + a$	If $a > 0$, then the translation _____. If $a < 0$, then the translation _____.		
$y = f(x + a)$	If $a > 0$, then the translation _____. If $a < 0$, then the translation _____.		
$y = a \cdot f(x)$	If $a > 0$, then there is a vertical _____. If $0 < a < 1$, then there is a vertical _____.		
$y = f(a \cdot x)$	If $a > 0$, then there is a horizontal _____. If $0 < a < 1$, then there is a horizontal _____.		
$y = -f(x)$	Vertical reflection over the _____.		
$y = f(-x)$	Horizontal reflection over the _____.		

2. For each leftover graph, glue the graphs into the spaces below and then write the equation that best represents each one.

Let's turn our attention to the function notation we first learned in Module 3 Unit 5. For each abstract form of the equation below, circle all of the transformations that must occur to the original function $f(x)$.

Abstract.

3. $g(x) = 3f(x) + 1$

Vertical Transformations	Horizontal Transformations
Stretch by 3 ✓	Stretch by 3
Shrink (compress) by $\frac{1}{3}$	Shrink (compress) by $\frac{1}{3}$
Translate (shift) up 1 ✓	Translate (shift) left 1
Translate (shift) down 1	Translate (shift) right 1

Explicit

$g(x) = 3x^2 + 1$

$y = x^2$

Abstract

4. $h(x) = -f(x - 2) - 3$

Explicit

$h(x) = -(x-2)^2 - 3$

Vertical Transformations	Horizontal Transformations
Reflect over the x-axis ✓	Reflect over the y-axis
Translate (shift) up 2	Translate (shift) left 2
Translate (shift) down 2	Translate (shift) right 2 ✓
Translate (shift) up 3	Translate (shift) left 3
Translate (shift) down 3 ✓	Translate (shift) right 3

5. Create your own equation with the following requirements:

Abstract

$g(x) = -f(3x) - 2$

Explicit

$g(x) = -(3x)^2 - 2$

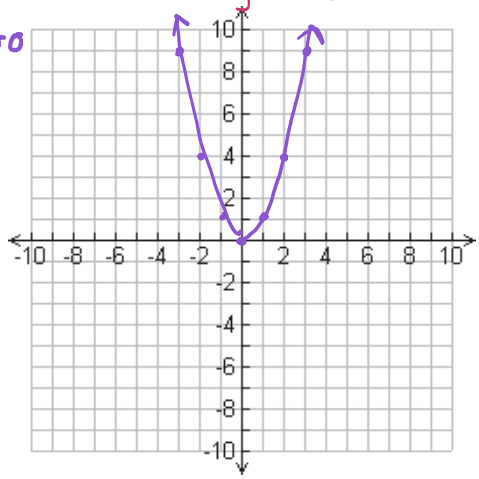
Vertical Transformations	Horizontal Transformations
Reflect over the x-axis	Reflect over the y-axis
Stretch by 3	Stretch by 3
Shrink (compress) by $\frac{1}{3}$	Shrink (compress) by $\frac{1}{3}$
Translate (shift) up 1	Translate (shift) left 1
Translate (shift) down 1	Translate (shift) right 1
Translate (shift) up 2	Translate (shift) left 2
Translate (shift) down 2	Translate (shift) right 2

Let's look at this idea of abstract equations with tables and graphs. We'll start with the parent graph of quadratic functions, $f(x) = x^2$.

6. Complete the table and graph the parent function.

$y = ax^2 + bx + c$
$y = a(x-h)^2 + k$

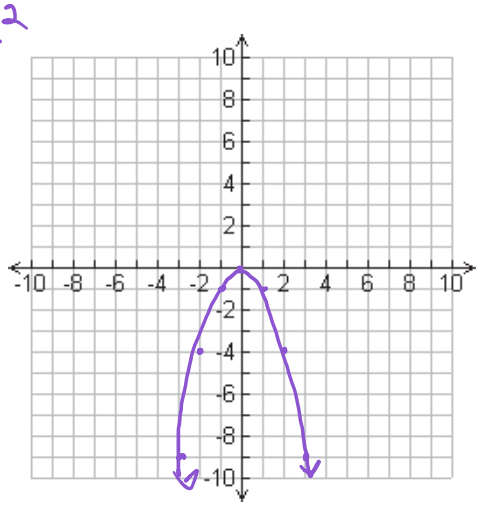
x	$f(x) = x^2$ → $f(x) = (x-0)^2 + 0$
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9



7. A. Complete the table and graph the function $g(x) = -f(x)$. Then in boxed Part D, write the explicit equation.

x	$g(x) = -f(x)^2$
−3	$-(-3)^2 = -9$
−2	$-(-2)^2 = -4$
−1	$-(-1)^2 = -1$
0	0
1	−1
2	−4
3	−9

$g(x) = -x^2$



B. What type of transformation is this?

Reflection over x-axis

C. How did the coordinate points change?

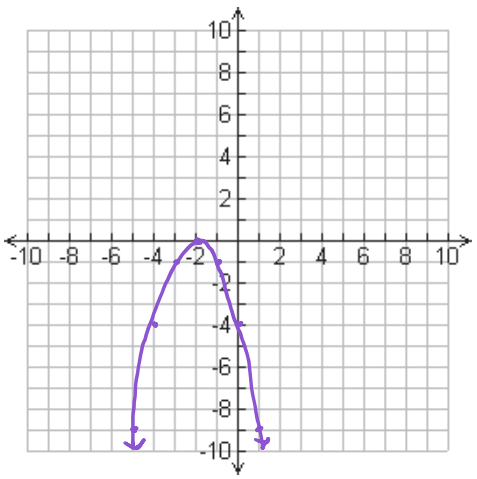
D. Explicit equation:

$g(x) = -x^2$

8. A. Complete the table and graph the function $h(x) = -f(x + 2)$. Then in boxed Part D, write the explicit equation.

$h(x) = -(x+2)^2$

x	$h(x) = -f(x + 2)$
−3	$-(-3+2)^2 = -1$
−2	$-(-2+2)^2 = 0$
−1	$-(-1+2)^2 = -1$
0	$-(0+2)^2 = -4$
1	−9
2	−16
3	−25



B. What type of transformation is this?

- horizontal shift left 2
- Reflect over x-axis

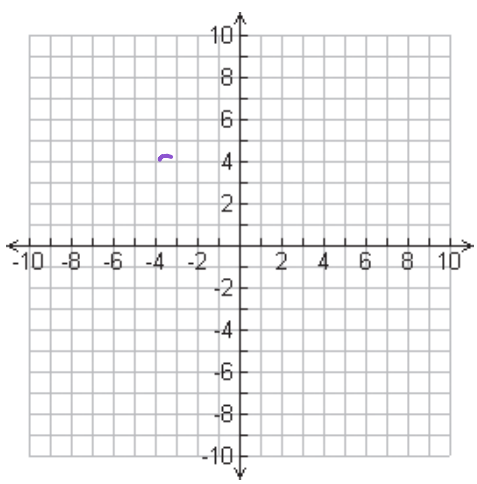
C. How did the coordinate points change?

D. Explicit equation:

$h(x) = -(x+2)^2$

9. A. Complete the table and graph the function $j(x) = -f(x + 2) + 3$. Then in boxed Part D, write the explicit equation.

x	$j(x) = -f(x + 2) + 3$
−3	
−2	
−1	
0	
1	
2	
3	



B. What type of transformation is this?

- horizontal left 2
- reflect over x-axis
- vertical up 3

C. How did the coordinate points change?

D. Explicit equation:

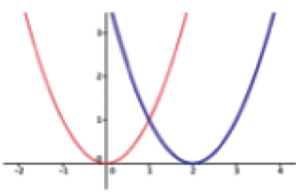
$j(x) = -(x+2)^2 + 3$

10. In each blank box, write the transformation that has occurred. One has been done for you.

Lesson Summary

Translations of Quadratic Functions
$= a(x - h)^2 + k$

Horizontal Translations



$f(x) = x^2$

$f(x - 2) = (x - 2)^2$

$f(x - h) = (x - h)^2$

Translate right h units

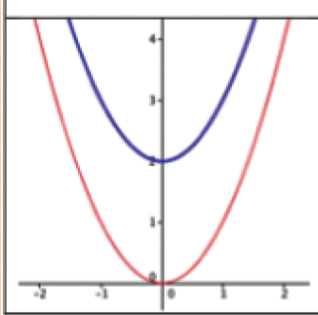
$f(x) = x^2$

$f(x + 2) = (x + 2)^2$

$f(x + h) = (x + h)^2$

Translate left

Vertical Translations

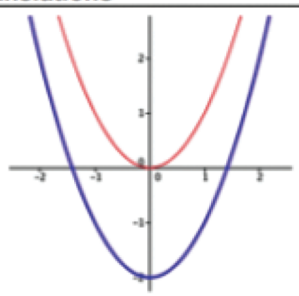


$f(x) = x^2$

$f(x) = x^2 + 2$

$f(x) = x^2 + k$

Translate up k.



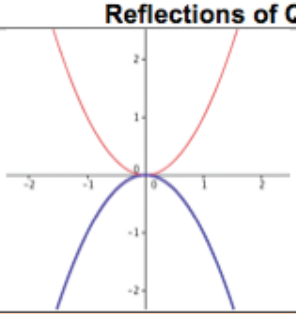
$f(x) = x^2$

$f(x) = x^2 - 2$

$f(x) = x^2 - k$

Translate down k

Reflections of Quadratic Functions



$f(x) = x^2$

$f(x) = -x^2$

Reflect over x-axis

Dilating: Stretching and Compressing Quadratic Functions

~~Stretching~~ shrinking

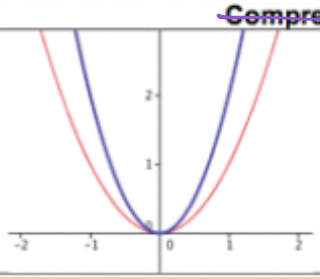
$f(x) = x^2$

$f(x) = \frac{1}{2}(x)^2$

$f(x) = a(x)^2$

vertical shrink

~~Compressing~~ stretching



$f(x) = x^2$

$f(x) = 2(x)^2$

$f(x) = a(x)^2$

vertical stretch

Source: Utah Electronic High School https://share.ehs.uen.org/taxonomy/term/188?page=2

NAME: _____ PERIOD: _____ DATE: _____

Homework Problem Set

1. Compare and contrast the graphs of the quadratic equations $y = x^2 + 1$ and $y = -2x^2 + 1$.

2. Compare and contrast the graphs of the quadratic equations $y = (x - 3)^2 + 2$ and $y = 2(x - 3)^2 + 4$.

3. Compare and contrast the graphs of the quadratic equations $y = (x + 5)^2$ and $y = (x - 5)^2$.

4. Compare and contrast the graphs of the quadratic equations $y = 3x^2$ and $y = \frac{1}{2}x^2$.

5. Write a quadratic equation with a vertical stretch of 7 and a vertex of (0, 3).

6. Write a quadratic equation with a vertical shrink (compression) of $\frac{1}{3}$ and a vertex of (3, 0).

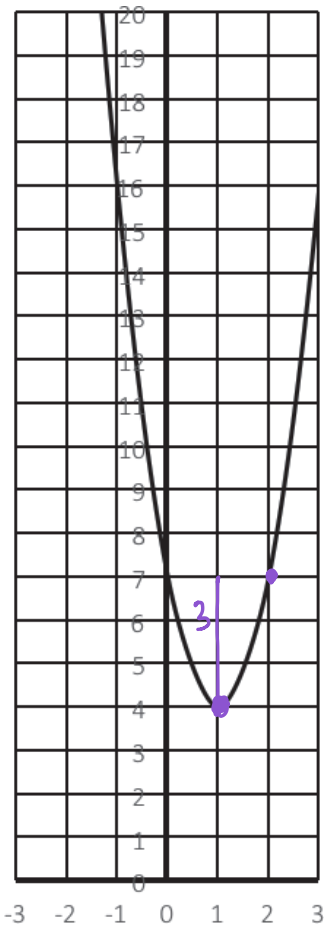
7. Use the number bank below to fill in the quadratic equation frame to describe the graph given.

$y = \underline{3}\,(x - \underline{1})^2 + \underline{4}$

Number Bank

0	1	2
3	4	5
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
−1	−2	−3

1, 3, 5
↓
3, 9, 15



(1,4)
X=1

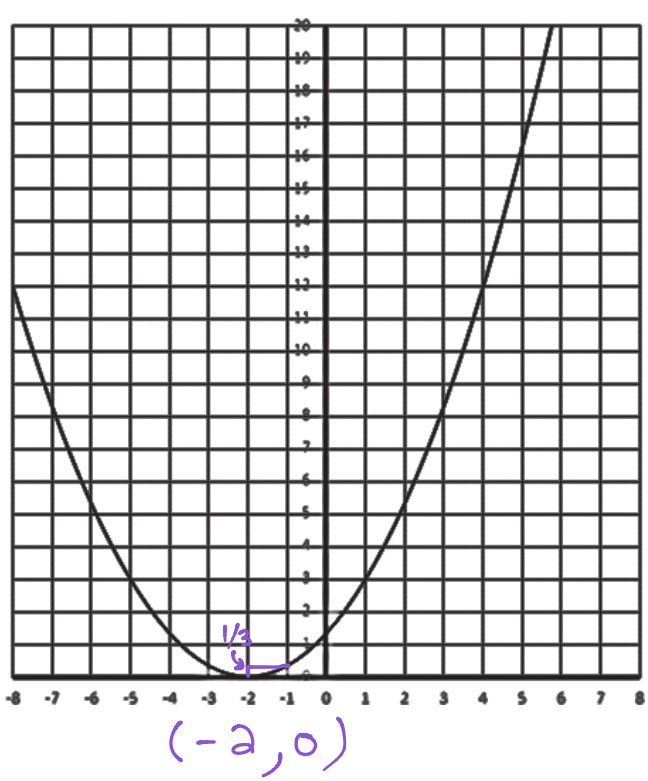
8. Use the number bank below to fill in the quadratic equation frame to describe the graph given.

(-2,0)

$y = \frac{1}{3}\,(x + \underline{2})^2 + \underline{0}$

Number Bank

0	1	2
3	4	5
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
−1	−2	−3

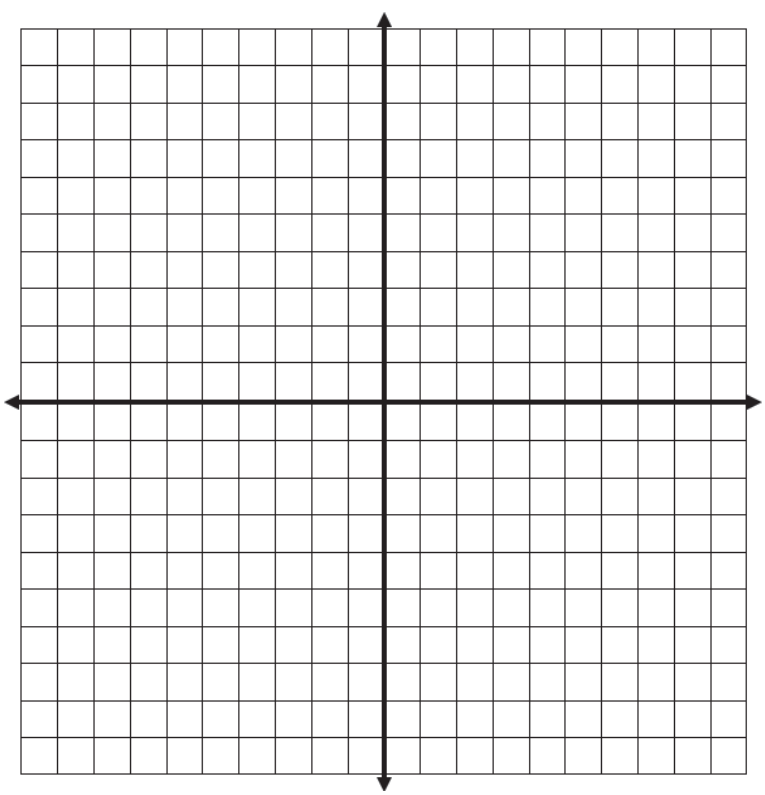


(-2,0)

9. $g(x) = -\frac{1}{3}f(x+4)$

Vertical Transformations	Horizontal Transformations
Reflect over the *x*-axis	Reflect over the *y*-axis
Translate (shift) up 4	Translate (shift) left 4
Translate (shift) down 4	Translate (shift) right 4
Stretch by 3	Stretch by 3
Shrink (compress) by $\frac{1}{3}$	Shrink (compress) by $\frac{1}{3}$

10. Graph $f(x) = (x-1)^2$ and $g(x) = -(x-1)^2 + 2$ on the grid below. Then describe the differences between the two graphs.



For Problems 11–14, write the abstract form of each equation if $f(x) = x^2$.

11. $g(x) = -(x-1)^2 + 2$

12. $h(x) = 3(x-1)^2 - 4$

13. $j(x) = -\frac{1}{8}(x-1)^2$

14. $k(x) = (x-2)^2$

Challenge for Problems 15–18, write the abstract form of each equation if $f(x) = (x-1)^2$.

15. $g(x) = -(x-1)^2 + 2$

16. $h(x) = 3(x-1)^2 - 4$

17. $j(x) = -\frac{1}{8}(x-1)^2$

18. $k(x) = (x-2)^2$