Honors Algebra 1B Name Key Per___

Unit 5 Test Review

Functions, Exponents, Radicals, Arithmetic Sequences, & Geometric Sequences

1. Given $f(x) = 6x - 4$ find:

a) $f(-2)$ $= 6(-2)-4$
$= -12-4$
$= -16$

b) $f(3)$ $= 6(3)-4$
$= 18-4$
$= 14$

2. Find the **domain** and **range** for each (interval notations). Determine which of the following are **functions** and explain your decision.

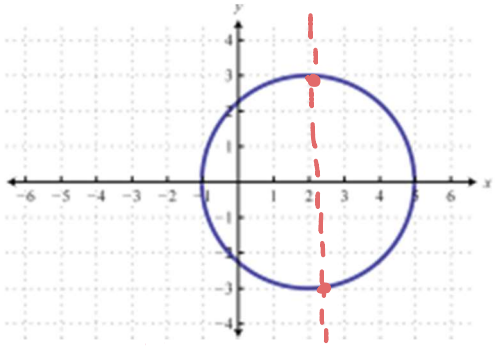
a) {(0, 2), (2, 4), (0, -3), (5, 4)}

Domain: D: {0, 2, 5}
Range: R: {-3, 2, 4}
Function: Yes No
Reason: The x-value 0 repeats

b) {(-6, 5), (6, 5), (5, 2), (4, 2)}

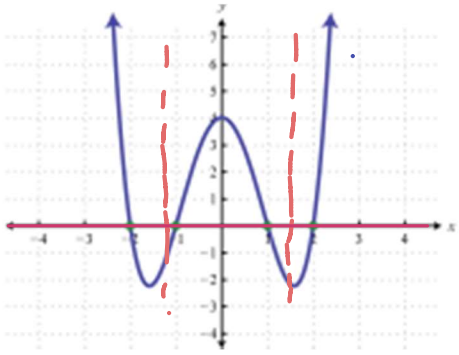
Domain: D: {-6, 4, 5, 6}
Range: R: {2, 5}
Function: Yes No
Reason: one x to one y

c)



Domain: D: [-1, 5]
Range: R: [-3, 3]
Function: Yes No
Reason: does not pass vertical line test

d)

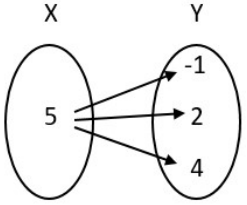


Domain: D: $(-\infty, \infty)$
Range: R: $[-2.25, \infty)$ approx.
Function: Yes No
Reason: passes vertical line test.

e) $f(x) = -2x + 5$

Domain: D: $(-\infty, \infty)$
Range: R: $(-\infty, \infty)$
Function: Yes No
Reason: It's a linear equation

f)



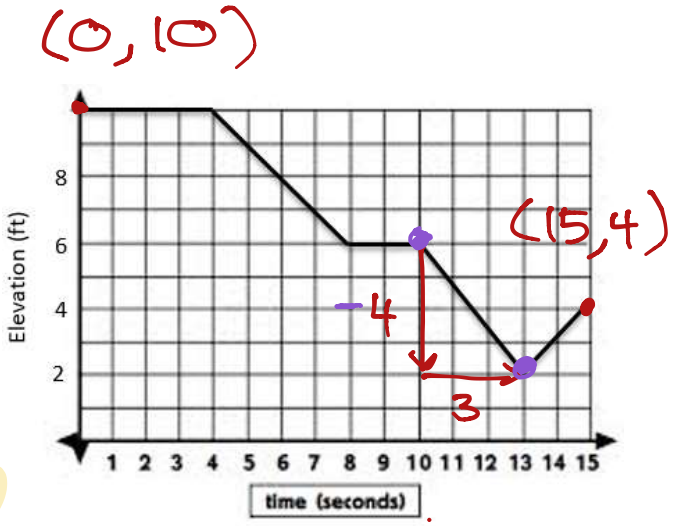
Domain: D: {5}
Range: R: {-1, 2, 4}
Function: ~~Yes~~ No
Reason: The input of 5 has more than one output

$m = \frac{y_2 - y_1}{x_2 - x_1}$

3. Use the graph on the right to answer the following questions.

a. Write a story of what this graph might be showing.

Answers will vary.



b. What is the average rate of change of elevation between 10 and 13 seconds? Leave answer as reduced fraction.

(10,6) (13,2)

$m = \frac{2-6}{13-10} = \frac{-4}{3}$

$m = -\frac{4}{3}$ ft/sec

c. What is the average rate of change of elevation between 0 and 15 seconds? Leave answer as reduced fraction

(0,10) (15,4)

$m = \frac{4-10}{15-0} = \frac{-6}{15} = \frac{-2}{5}$

$m = -\frac{2}{5}$ ft/sec

d. State the domain and range for this graph in interval notations and inequalities.

Domain: $[0,15]$ Range: $[2,10]$

4. Simplify each expression so that there are no negative exponents.

a) $\frac{n^3 m}{n^{-1} m^2} = \frac{n^3 \cdot n \cdot m}{m^2}$

$n^4 m^{-1} = \frac{n^4}{m} = \frac{n^4}{m^2} = \frac{n^4}{m}$

b) $\frac{(ab^3)^2}{a}$

$\frac{a^2 b^6}{a} = ab^6$

$\left(\frac{2^1 a^2 b^{-3}}{5^1 a^{-4} b^1}\right)^{-2}$

$\frac{2^{-2} a^{-4} b^6}{5^{-2} a^8 b^{-2}}$

$\frac{25 b^8}{4 a^{12}}$

c) $\frac{x^{-2} y^3 z}{x^0 y z^4}$

$x^{-2} y^2 z^{-3} = \frac{y^2}{x^2 z^3}$

d) $\left(\frac{n^{-1} m^{-2}}{n^{-2} m^{-3}}\right)^{-1}$

$\frac{nm^2}{n^2 m^3} = n^{-1} m^{-1} = \frac{1}{nm}$

5. Simplify the radical expressions as much as possible

a) $\sqrt{75x^3}$

$\sqrt{25 \cdot 3 \cdot x \cdot x \cdot x}$

$5x\sqrt{3x}$

b) $\sqrt{162a^2b^5c}$

$\sqrt{81 \cdot 2\, a\, a\, b\, b\, b\, b\, b\, c}$

$9|a|b^2\sqrt{2bc}$

c) $\sqrt[3]{32n^5m^9}$

$\sqrt[3]{8 \cdot 4\, n\, n\, n\, n\, n\, m\, m\, m\, m\, m\, m\, m\, m}$

$2nm^3\sqrt[3]{4n^2}$

#6-11: State whether the following sequences are arithmetic, geometric, or neither, then state the common difference (d) or ratio (r) if applicable.

6. 6, 24, 96, 384, …

×4 ×4 ×4

Geometric

$r = 4$

7. 3, 14, 25, 36, ...

+11 +11 +11

Arithmetic

$d = 11$

8. 1, 2, 6, 24, …

neither

9. 1, 3, 7, 13, ...

neither

10. $-\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \frac{1}{32}, \ldots\ldots$

Geometric

$r = -\frac{1}{2}$

11. -7, -16, -25, …

-9 -9

Arithmetic

$d = -9$

#12-13: State whether the following formulas are recursive or explicit. Write the first five terms for each sequence. Show all work.

12. $f(n) = -2n^2 + 1$

Explicit -1, -7, -17, -31, -49

$f(1) = -2(1)^2 + 1 = -1$
$f(2) = -2(2)^2 + 1 = -7$
$f(3) = -2(3)^2 + 1 = -17$
$f(4) = -2(4)^2 + 1 = -31$
$f(5) = -2(5)^2 + 1 = -49$

13. $f(n+1) = 2f(n) - 6$ and $f(1) = 2$ for $n \geq 1$

Recursive 2, -2, -10, -26, -58

$f(1) = 2$
$f(2) = 2f(1) - 6 = 2(2) - 6 = -2$
$f(3) = 2f(2) - 6 = 2(-2) - 6 = -10$
$f(4) = 2f(3) - 6 = 2(-10) - 6 = -26$
$f(5) = 2f(4) - 6 = 2(-26) - 6 = -58$

#14-17: State whether each sequence is arithmetic or geometric then write a general term (**explicit**) formula for each sequence and find the **10th term**.

14. 30, 27, 24, 21, …

Arithmetic $f(1) = 30$, $d = -3$

$f(n) = 30 - 3(n-1)$
$= 30 - 3n + 3$
$= -3n + 33$
$f(10) = -3(10) + 33 = 3$

15. $5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots\ldots$

Geometric: $f(1) = 5$, $r = -\frac{1}{5}$

$f(n) = 5\left(-\frac{1}{5}\right)^{n-1}$
$f(10) = 5\left(-\frac{1}{5}\right)^{10-1}$
$= -0.00000256$

16. -2, -6, -18, …

Geometric: $f(1) = -2$, $r = 3$

$f(n) = -2(3)^{n-1}$
$f(10) = -2(3)^{10-1}$
$= -2(3)^9$
$= -39366$

17. -5, 16, 37, …

Arithmetic: $f(1) = -5$, $d = 21$

$f(n) = -5 + 21(n-1)$
$= -5 + 21n - 21$
$= 21n - 26$
$f(10) = 21(10) - 26 = 184$

#18-20: For each word problem below, write the explicit formula that represents the situation then answer the question.

18. You want to buy a new TV for $2100, but you have to save up over time. You save $25 in the first month, $35 the second month, $45 the third month and so on. How much will you saved in the 12th month?

25, 35, 45, ... $f(1)=25$, $d=10$

$f(n)=25+10(n-1)$

$=25+10n-10$

$=10n+15$

$f(12)=10(12)+15$

$=120+15$

$=135$

General term formula: $f(n)=10n+15$ Answer: $135

19. Edgar is getting better at math. On his first quiz he scored 57 points, then he scores 61 and 65 on his next two quizzes. If his scores continued to increase at the same rate, what will be his score on his 9th quiz? Show all work.

57, 61, 65, ... $f(1)=57$ $d=4$

$f(n)=57+4(n-1)$

$=57+4n-4$

$=4n+53$

$f(9)=4(9)+53$

$=36+53$

$=89$

General term formula: $f(n)=4n+53$ Answer: 89 points

20. Kevin is trying to reduce the amount of sugar he eats each day. In the first day, he allows himself to have a maximum of 50 grams. Each day he will reduce his maximum sugar intake by 3 grams. How many grams will he be allowed to have at the end of 2 weeks?

50, 47, 44, 41, ... $f(1)=50$, $d=-3$

$f(n)=50-3(n-1)$

$=50-3n+3$

$=-3n+53$

$f(14)=-3(14)+53$

$=-42+53$

$=11$

General term formula: $f(n)=-3n+53$ Answer: 11 grams

21. If groceries now cost Mrs. Brooks $240 per week, she predicts that the cost will increase 10% per year due to inflation and her three growing boys eating more and more each year. Her oldest boy, John, is trying to figure out how much money will she be spending per week on groceries after 5 years. Below is his work – find and correct his mistake(s). Explain your reasoning.

$f(n) = 240(.90)^{n-1}$

$f(5) = 240(.90)^{5-1}$

She will be spending $157.46

* $r = 110\% = 1.1$ (100% + 10%)
* After 5 years → $n = 6$

$f(n) = 240(1.10)^{n-1}$

$f(6) = 240(1.10)^{6-1}$

$= 240(1.10)^5$

$= 386.52$

22. A radioactive element has a half-life of 2 days. This means only half of the material is left after 2 days. If there are 10,240 grams present currently, then how much will be present at the beginning of the 16th day?

$f(1) = 10240 \quad r = \frac{1}{2}$

16 days → $n = 8$ (half-life cycles)

$f(n) = 10240\left(\frac{1}{2}\right)^{n-1}$

$f(8) = 10240\left(\frac{1}{2}\right)^{8-1} = 80$

General term formula: $f(n) = 10240\left(\frac{1}{2}\right)^{n-1}$ Answer: 80 grams

23. Consider the arithmetic sequence 27, 13, -1, …

$f(1) = 27 \quad d = -14$

a. Find an explicit rule for the sequence in terms of n.

$f(n) = 27 - 14(n-1)$

$= 27 - 14n + 14$

$= -14n + 41$

n^{th}?

27, 13, -1, …, -841

b. Find the 40th term.

$f(40) = -14(40) + 41$

$= -519$

c. If the nth term is -841, find the value of n.

$-841 = -14n + 41$

$-41 \qquad -41$

$-882 = -14n$

$63 = n$

24. Arithmetic sequences are modeled by linear functions and use the explicit formula: $f(n) = f(1) + d(n-1)$

Geometric sequences are modeled by exponential function and use the explicit formula: $f(n) = f(1) \cdot r^{n-1}$

25. Graphs of sequences are discrete or continuous? Explain.

Discrete, terms can only be integers.

26. Given the arithmetic sequence: 52, a, b, 64. Find the value of a•b.

$f(1) = 52$

$f(4) = 64$

$f(n) = f(1) + d(n-1)$

$f(4) = f(1) + d(4-1)$

$64 = 52 + d(3)$

$12 = 3d$

$4 = d$

$52, \underset{a}{56}, \underset{b}{60}, 64$

56×60

$\boxed{3360}$

$\boxed{f(4) = 64}$

$f(4) = f(1) + d(4-1)$

$64 = 52 + d(3)$

$-52 \quad -52$

$12 = 3d$

$4 = d$

Answer Key:

1a. -16	**1**b. 14	**2**a. D: {0, 2, 5} R: {-3, 2, 4} No Repeated X	**2**b. D: {-6, 4, 5, 6} R: {2, 5} Yes One X to one Y
2c. D: [-1, 5] R: [-3, 3] No Fails Vertical Line Test	**2**d. D: (-∞, ∞) R: [-2, ∞) Yes Passes Vertical Line Test	**2**e. D: (-∞, ∞) R: [-∞, ∞) Yes Linear	**2**f. D: {5} R: {1, 2, 4} No Repeated X values
3a. [stories will vary]	**3**b. $-\frac{4}{3}$ ft / sec	**3**c. $-\frac{2}{5}$ ft / sec	**3**d. Domain: [0,15] Range: [2,10]
4. **a)** $\frac{n^4}{m}$ **b)** ab^6 **c)** $\frac{y^2}{x^2z^3}$ **d)** $\frac{1}{nm}$	**5**. **a)** $5x\sqrt{3x}$ **b)** $9a^2b^2\sqrt{2bc}$ **c)** $2nm^3\sqrt[3]{4n^2}$	**6**. geometric, r = 4	**7**. arithmetic, d = 11
8. neither	**9**. neither	**10**. geometric, r = -1/2	**11**. arithmetic, d = -9
12. explicit, -1, -7, -17, -31, -49	**13**. recursive, 2, -2, -10, -26, -58	**14**. $f(n) = -3n + 33$ f(10) = 3	
15. $f(n) = 5\left(-\frac{1}{5}\right)^{n-1}$ f(10) = $= -\frac{1}{390625} = -0.00000256$	**16**. $f(n) = -2(3)^{n-1}$ f(10) = -39366	**17**. $f(n) = 21n - 26$ f(10) = 184	**18**. $f(n) = 25 + 10(n-1)$ f(12) = \$135
19. $f(n) = 57 + 4(n-1)$ f(9) = 89 points	**20**. $f(n) = -3n + 53$ f(14) = 11 grams	**21**. two mistakes: correct answer is \$386.52	**22**. $f(n) = 10240\left(\frac{1}{2}\right)^{n-1}$ f(8) = 80 grams
23a. $f(n) = -14n + 41$	**23**b. f(40) = -519	**23**c. n = 63	**24**. linear, $f(n) = f(1) + d(n-1)$ exponential, $f(n) = f(1) \cdot r^{n-1}$
25. discrete	**26**. 3360		

$f(n) = 2f(n-1) + 5$

for $f(1) = -3$

Write the first 4 terms

$f(1) = -3$

$f(2) = \underline{-1} = 2(-3) + 5$

$f(3) = \underline{3} = 2(-1) + 5$

$f(4) = \underline{11} = 2(3) + 5$