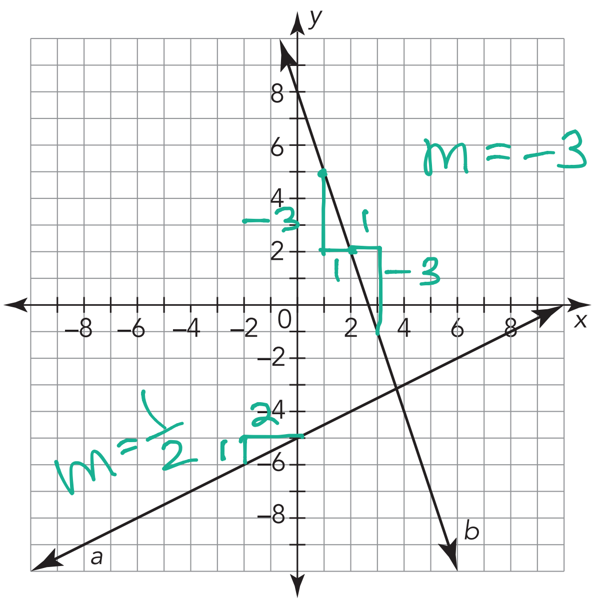
At the Arcade

2

Linear Relationships in Tables

WARM UP

Use similar right triangles to determine the slope of each line.



LEARNING GOALS

- Determine the rate of change of a linear relationship by reading (*x*, *y*) values from a table.
- Develop a formula to calculate the slope of a line given a table of values.
- Use the slope formula to calculate the rate of change from a table of values or two coordinate pairs.
- Determine whether a table of values represents a linear proportional or linear non-proportional relationship.

KEY TERM

- first differences

You have used graphs to analyze and compare linear relationships. You have used similar right triangles to determine slopes of lines graphed on a coordinate plane. How can you calculate the slope of a linear relationship given a table of values without creating a graph?

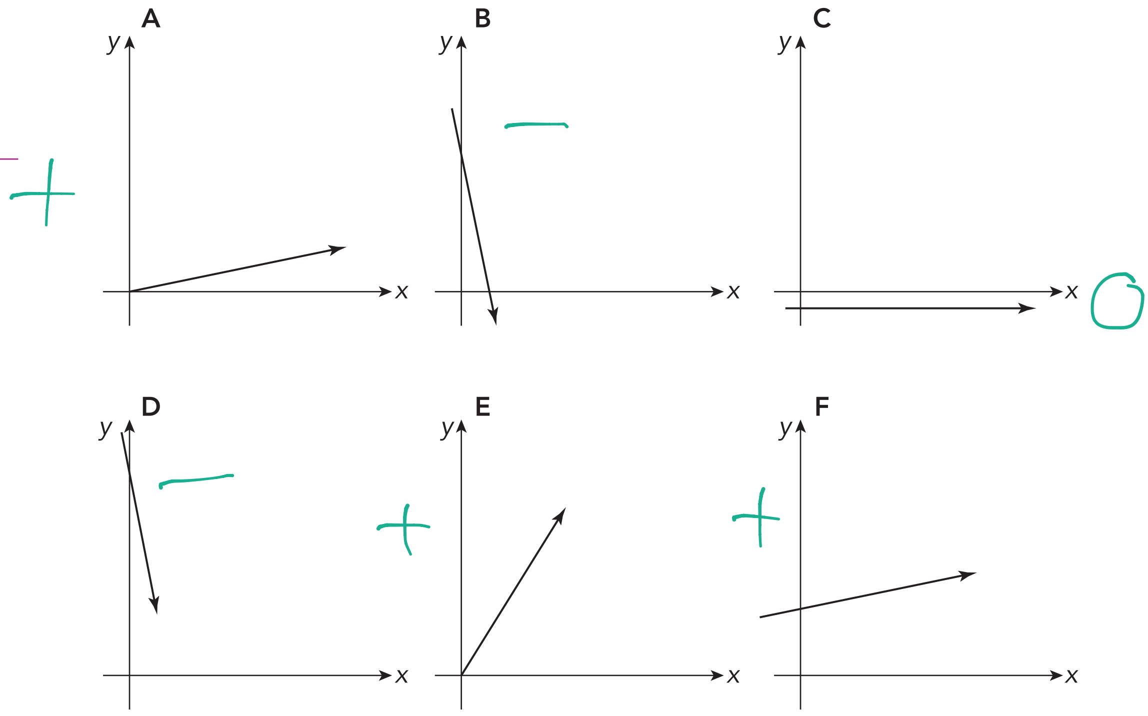
Getting Started

Slope Matching

Remember, the rate of change, or slope, of a line represents a ratio of the change in the dependent quantity to the change in the independent quantity.

You have used slope to describe the steepness and direction of a line. Consider each graph shown.

1. Identify the graph(s) whose line may have the given slope. Then, describe your strategy for matching the graphs to the given slopes.



a. $\frac{1}{4}$ A, E, F

b. 0 C

c. $\frac{5}{4}$ A, E, F

d. −3 B, D

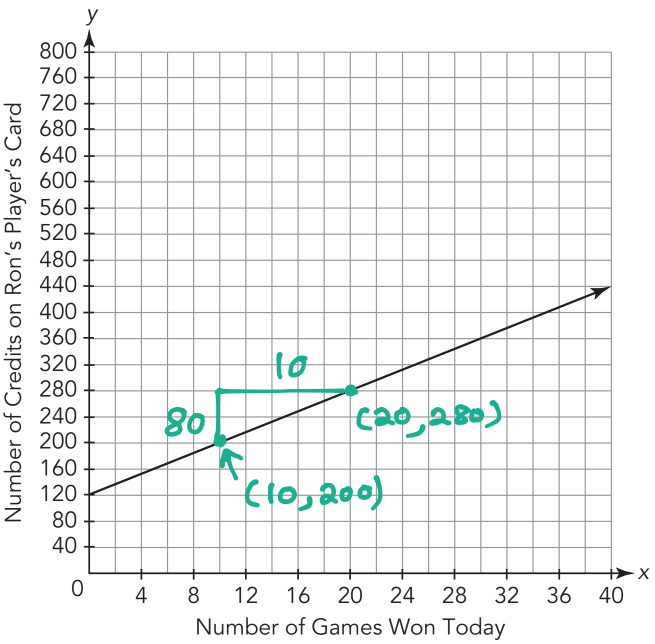
2. How did you use the graphs to estimate their slope?

ACTIVITY 2.1

Analyzing a Linear Relationship from a Table

Ron has a player's card for the arcade at the mall. His player's card keeps track of the number of credits he earns as he wins games. Each winning game earns the same number of credits, and those credits can be redeemed for various prizes. Ron has been saving his credits to collect a prize worth 500 credits.

The table and graph show the number of credits Ron had on his game card at various times today when he checked his balance at the arcade.



Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
b 0	120
12	216
18	264
25	320
40	440

1. **Is this relationship proportional or non-proportional? Explain how you know.**

non-proportional, not through (0,0)

2. **Explain the meaning of the ordered pair (0, 120) listed in the table.**

b

The starting credits Ron had.

3. **Use the graph to determine the slope of the line. Then explain the meaning of the slope in terms of this problem situation.**

$m = \frac{80}{10} = \frac{8}{1}$ * For every game Ron won, he earned 8 credits

NOTES

4. Analyze Rhonda's reasoning. Explain why her reasoning is incorrect.

Rhonda

$$\frac{440 \text{ credits}}{40 \text{ games won}} = \frac{11 \text{ credits}}{1 \text{ game won}}$$

The slope is 11.

440-120

$$\frac{320}{40} = \frac{32}{4} = \frac{8}{1}$$

5. Before Ron started winning games today, how many games had he won for which he had saved the credits on his player's card? Show your work.

$$\frac{120}{8} = 15 \text{ games}$$

6. After Ron won his fortieth game today, how many more games does he need to win to collect a prize worth 500 credits? Show your work and explain your reasoning.

$$y = mx + b$$
$$\downarrow$$
$$y = 8x + 120$$
$$\downarrow$$
$$(40)$$

$$8(40) = 320 + 120$$
$$= 440$$

$$\begin{array}{r} 500 \\ -440 \\ \hline 60 \end{array}$$

$60 \div 8 = 7.5$

≈ 8 games

7. Summarize what you know about this scenario based on your analysis. Be sure to include each item listed.

- the initial values of the independent and dependent variables in the context of the problem
- a sentence explaining the rate of change in terms of the context of the problem
- the final values of the independent and dependent variables in the context of the problem

ACTIVITY
2.2

Calculating Rate of Change from a Table

So far, you have determined the rate of change from a graph using similar triangles and writing a ratio of the vertical distance to the horizontal distance. However, you can also determine the rate of change, or slope, from a table.

1. Complete the steps to determine the slope from a table.

x y

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
0	120
12	216
18	264
25	320
40	440

$m = \frac{\text{vertical change}}{\text{horizontal change}}$

$m = \frac{?}{?}$

a. Choose any two values of the dependent variable. Calculate their difference. (y)

$320 - 120 = 200$ (vertical change)

b. Calculate the difference between the corresponding values (x) of the in dependent variable. It is important that the order of values you used for determining the difference of the independent variables be followed for the dependent variables.

$25 - 0 = 25$ (horizontal change)

c. Write a rate to compare the change in the dependent variable to the change in the independent variable.

$m = \frac{200}{25}$

d. Rewrite the rate as a unit rate.

$\frac{200 \div 25}{25 \div 25} = \boxed{\frac{8}{1}}$

2. Examine each example. Follow the arrows to calculate the slope. Was the slope calculated correctly in each case? Explain any errors that may have occurred when the arrows were drawn.

Example 1

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
0	120
12	216
18	264
25	320
40	440

Example 2

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
0	120
12	216
18	264
25	320
40	440

Example 3

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
0	120
12	216
18	264
25	320
40	440

There is a formal mathematical process that can be used to calculate the slope of a linear relationship from a table of values with at least two coordinate pairs.

The slope can be calculated using two ordered pairs and the formula:

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1},}$$

slope formula

where the first point is (x_1, y_1) and the second point is (x_2, y_2).

WORKED EXAMPLE

You can calculate the slope of a linear relationship from a table of values. Consider the table showing the number of credits Ron had on his game card at various times at the arcade.

Number of Games Ron Won Today	Number of Credits on Ron's Player's Card
0	120
12 x_1	216 y_1
18	264
25	320
40 x_2	440 y_2

Step 1: From the table of values, use (12, 216) as the first point and (25, 320) as the second point.

Step 2: Label the points with the variables.

$$\begin{array}{cc} (12,\ 216) & (25,\ 320) \\ \downarrow\ \ \downarrow & \downarrow\ \ \downarrow \\ (x_1,\ y_1) & (x_2,\ y_2) \end{array}$$

Step 3: Use the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{320 - 216}{25 - 12} \\ &= \frac{104}{13} \\ &= 8 \end{aligned}$$

The slope is $\frac{8 \text{ credits}}{1 \text{ game}}$ or 8 credits per game.

3. Repeat the process to calculate the slope using two different values from the table. Show your work.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{440 - 216}{40 - 12} = \frac{224 \div 28}{28 \div 28} = \boxed{\frac{8}{1}}$$

4. How is using the slope formula given a table related to using similar triangles given a graph?

$m = \frac{y_2 - y_1}{x_2 - x_1}$

ACTIVITY 2.3 Practice with Linear Relationships in Tables

Analyze the values in the table before you start calculating the rate of change. Do you think the rate of change will be positive or negative?

You can now use the slope formula to calculate the slope of a line given a table of values.

1. Calculate the slope of each linear relationship using the formula. Show all your work.

a.

Number of Carnival Ride Tickets	Cost (dollars)
x_1 4	y_1 9
8	12
x_2 16	y_2 18
32	30

$m = \frac{18-9}{16-4} = \frac{9}{12} = \boxed{\frac{3}{4}}$

b.

x	y
x_2 −1	y_2 13
x_1 0	y_1 −2
4	−62
10	−152

$m = \frac{13-(-2)}{-1-0} = \frac{13+2}{-1} = \frac{15}{-1} = \boxed{-15}$

c.

Days Passed	Vitamins Remaining in Bottle
x_1 7	y_1 25
8	23
9	21
x_2 10	y_2 19

$m = \frac{19-25}{10-7}$

$m = \frac{-6}{3} = \boxed{-2}$

d.

x	y
x_1 7	y_1 9
x_2 18	y_2 9
29	9
40	9

$m = \frac{9-9}{18-7} = \frac{0}{11} = \boxed{0}$

e. (10, 25) and (55, 40)

$x_1 \; y_1 \quad x_2 \; y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40-25}{55-10}$

$= \frac{15 \div 5}{45 \div 5} = \frac{3}{9} = \boxed{\frac{1}{3}}$

f. (4, 19) and (24, 3)

$x_1 \; y_1 \quad x_2 \; y_2$

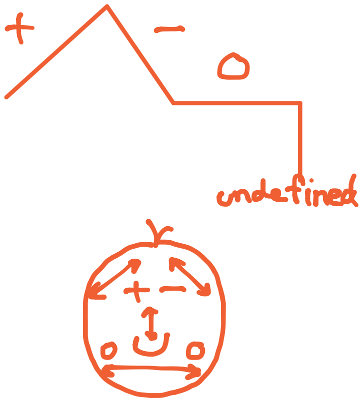
$m = \frac{3-19}{24-4} = \frac{-16}{20}$

$= \boxed{-4}$

45÷5 9 $\boxed{5}$ $\boxed{5}$

2. Which relationships in Question 1 are proportional relationships? Explain your reasoning. (0,0)

None passes through (0,0)

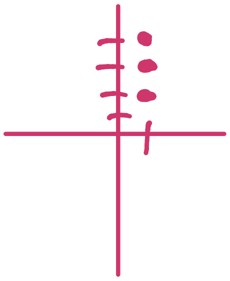


3. Complete each sentence to describe how you can tell whether the slope of a line is positive or negative by analyzing given points.

   a. If the slope of a line is positive, then as the value of x increases the value of y increases. +

   b. If the slope of a line is negative, then as the value of x increases the value of y decreases. −

4. Consider the relationship represented in each table shown.

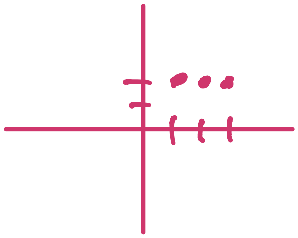


$x = 1$	
x	y
1	−5
1	10
1	15
1	30

vertical

$y = 2$	
x	y
5	2
6	2
7	2
8	2

horizontal



(1,2)
(1,3)
(1,4)

(1,2)
(2,2)
(3,2)

   a. Sketch a graph of each relationship. Which relationship is represented by a horizontal line? a vertical line?

$x=1$ $y=2$

   b. What can you conjecture about the slopes of these lines?

undefined $m=0$

$m = \frac{\#}{0}$ $m = \frac{0}{\#} = 0$

ACTIVITY 2.4 Determining If a Relationship Is Linear

line.

You previously used similar right triangles to show that if you are given a line on a graph, then the slope is the same between any two points on that line. The converse is also true. If the slope between every ordered pair in a table of values is constant, then the ordered pairs will form a straight line.

always the same.

A conditional statement uses the words "if" and "then" to show assumptions and conclusions. For example, if today is Monday, then tomorrow is Tuesday. A converse statement switches the order. For example, if tomorrow is Tuesday, then today is Monday. For any conditional statement the converse may or may not be true.

So, in order to determine if a table of values represents a linear relationship, show that the slope is the same between every set of ordered pairs.

linear → same slope

1. Calculate the slope between the given ordered pairs to determine if they form a straight line. Show your work.

x	y
4	13
9	28
11	34
16	47

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$x_1\ y_1\quad x_2\ y_2$

a. (4, 13) and (9, 28)

$\frac{28-13}{9-4} = \frac{15}{5} = \boxed{\frac{3}{1}}$

$x_1\ y_1\quad x_2\ y_2$

b. (9, 28) and (11, 34)

$\frac{34-28}{11-9} = \frac{6}{2} = \boxed{\frac{3}{1}}$

$x_1\ y_1\quad x_2\ y_2$

c. (11, 34) and (16, 47)

$\frac{47-34}{16-11} = \boxed{\frac{13}{5}}$ ←

d. Will the ordered pairs listed in the table form a straight line when plotted? Explain your reasoning.

No, the slopes are not constant.

2. Determine whether the ordered pairs listed in each table will form a straight line when plotted. Show your work. Explain your reasoning.

a.

x	y
2	7
6	13
8	16
20	34

4, 2, 12

6, 3, 18

$\frac{6}{4} = \boxed{\frac{3}{2}}$ ✓ $\boxed{\frac{3}{2}}$ ✓ $\frac{18}{12} = \boxed{\frac{3}{2}}$ ✓

b.

x	y
1	33
2	40
3	47
4	54
5	61

1, 1, 1, 1

7, 7, 7, 7

$m = \frac{7}{1}$

* The slope is constant. The relationship is linear.

Consecutive means one right after the other, such as 12, 13, and 14.

When the values for the independent variable in a table are consecutive integers, you can examine only the column with the dependent variable and calculate the differences between consecutive values. If the differences are the same each time, then you know that the rate of change is the same each time. The relationship is a linear relationship.

WORKED EXAMPLE

The differences have been calculated for the table shown.

x	y
1	99
2	86
3	73
4	60
5	47

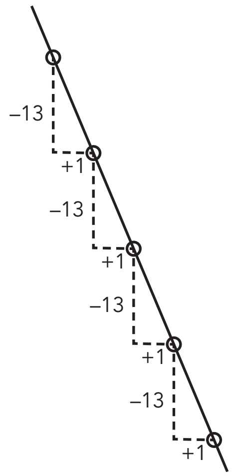
y_2 y_1

$86 - 99 = -13$

$73 - 86 = -13$

$60 - 73 = -13$

$47 - 60 = -13$



The differences between consecutive values for the dependent variable are the same each time. Therefore the rate of change is the same each time as well. The ordered pairs in this table will therefore form a straight line when plotted.

In this process, you are calculating *first differences*. **First differences** are the values determined by subtracting consecutive y-values in a table when the x-values are consecutive integers. The first differences in a linear relationship are constant.

3. Use first differences to determine whether the ordered pairs in each table represent a linear relationship. Show your work and explain your reasoning.

a.

x	y
1	25
2	34
3	45
4	52
5	61

1, 1, 1, 1

9, 11, 7, 9

Non-linear.

$\frac{9}{1}$ $\frac{11}{1}$ $\frac{7}{1}$ $\frac{9}{1}$

b.

x	y
1	12
2	8
3	4
4	0
5	−4

1, 1, 1, 1

−4, −4, −4, −4

$-\frac{4}{1}$ linear

c.

x	y
1	1
2	4
3	9
4	16
5	25

1, 1, 1, 1

3, 5, 7, 9

$\frac{3}{1}$, $\frac{5}{1}$, $\frac{7}{1}$, $\frac{9}{1}$

non-linear

d.

x	y
1	15
2	18
3	21
4	24
5	27

1, 1, 1, 1

3, 3, 3, 3

$\frac{3}{1}$

linear

NOTES

TALK the TALK

Walk the Walk

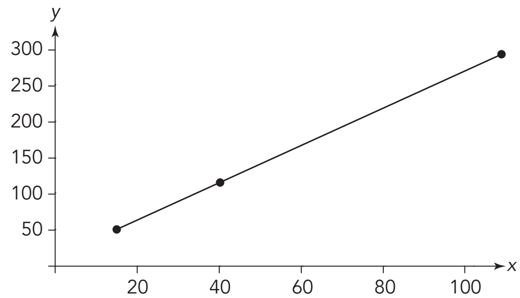
The table shows the distance Angel walked compared to the number of steps she took.

Number of Steps	Distance Walked (ft)
16	50
40	120
110	300

1. Calculate the slope between each set of ordered pairs. Show your work.

2. Is the graph of the relationship linear? What does this mean in terms of the problem situation?

3. The ordered pairs from the table are represented on the given graph. Show how to use the graph to verify the slope you calculated from the table.



4. How is calculating the slope from a table similar to calculating the slope of a linear relationship from a graph?

Assignment

Write

Define the term *first differences* in your own words.

Remember

You can use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to determine the rate of change between two points represented in a table of values. If the rate is constant, this formula gives the rate of change for the relationship, or slope. The slope of a horizontal line is 0. The slope of a vertical line is undefined.

Practice

1. Each table represents a linear relationship. Which table(s) represent a slope of 2?

Table 1

x	y
0	32
3	26
5	22
9	14

Table 2

x	y
1	3
2	5
3	7
4	9

Table 3

x	y
0	8
3	14
7	22
9	26

2. Calculate the rate of change between the points listed in each table. Determine if the table represents a proportional relationship.

a.

x	y
2	14
5	35
7	49
10	70

b.

x	y
−10	50
−2	10
4	−20
14	−70

c.

x	y
−1	−24
2	48
4	90
8	192

d.

x	y
−6	12
−3	6
3	−6
6	−10

e.

x	y
2	13.5
5	33.75
10	67.5
15	101.25

f.

x	y
−4	−38
−1	−9.5
2	19
3	27

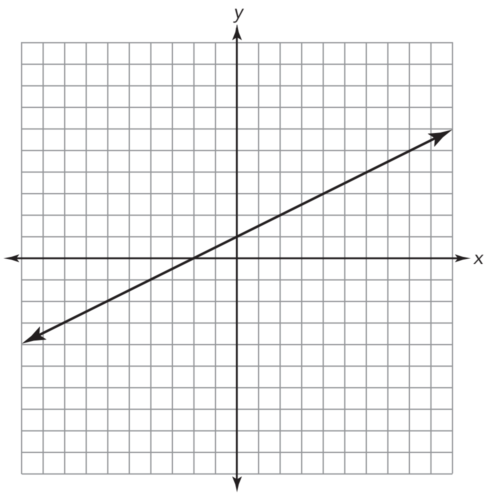
Stretch

Is the relationship described by the equation $y = x^2$ linear? Is it proportional? Describe how you determined your answers.

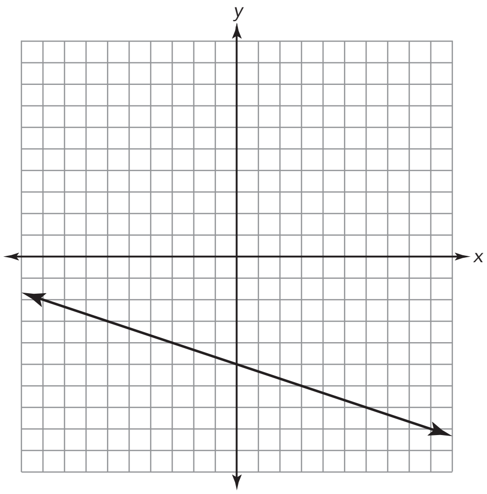
Review

1. Determine the slope of each linear relationship.

a.



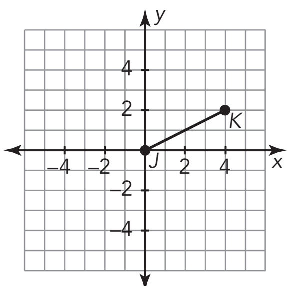
b.



c. $y = 2x$

d. $\frac{5}{6} = \frac{y}{x}$

2. Consider the graph shown.



a. Segment JK is rotated 90° clockwise resulting in segment $J'K'$. What are the coordinates of K'?

b. Segment JK is reflected across the line $x = -1$ resulting in segment $J'K'$. What are the coordinates of K'?