2

Rising, Running, Stepping, Scaling

Dilating Figures on the Coordinate Plane

WARM UP

Scale up or scale down to determine the value of the variable in each equivalent ratio.

1. $3 : 1 = 25.5 : z$
2. $2 : 5 = a : 30$
3. $1 : 4 = x : 80$
4. $9.9 : 10 = 99 : p$

LEARNING GOALS

- Dilate figures on a coordinate plane.
- Understand the dilation of a figure on the coordinate plane as a scaling up or scaling down of the coordinates of the figure.
- Describe how a dilation of a figure on a coordinate plane affects the coordinates of a figure.
- Distinguish between a dilation centered at the origin and a dilation not centered at the origin.

You have used transformations called dilations to create similar figures. How can you use coordinates to determine whether two figures are similar?

Getting Started

The Escalator or the Stairs

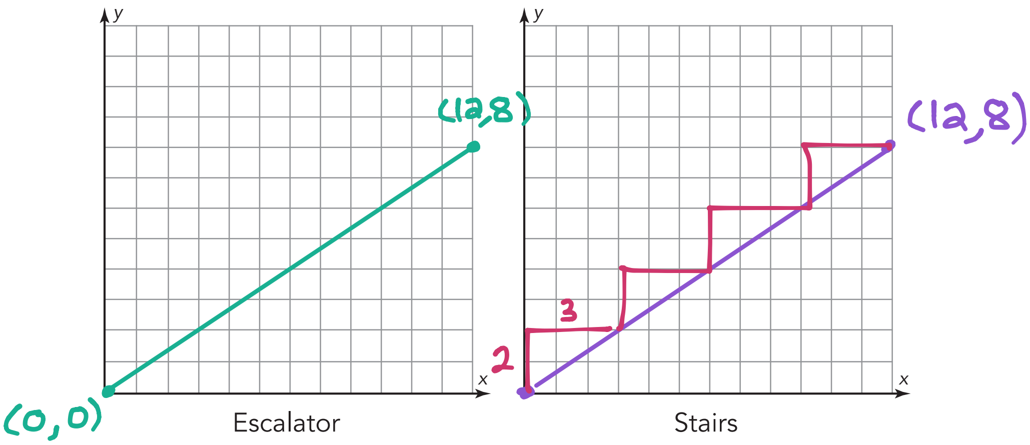
Bob is riding an escalator. The escalator starts at (0, 0) and drops Bob off at (12, 8).

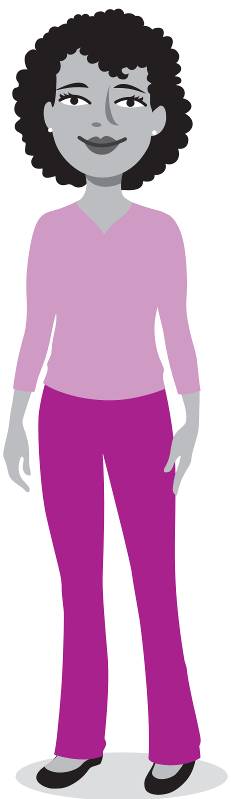
1. Use the coordinate planes given to represent Bob's journey.

a. Draw a line to show Bob's path on the escalator.

Think about equivalent ratios, scaling up, and scaling down.

b. Alice takes the stairs. Draw steps starting at the origin that will take Alice to the same location as Bob. Make all of the steps the same.





2. How is taking the stairs similar to riding the escalator? How is it different? Explain your reasoning.

– Escalator is a smooth path

– Steps are repeating ups and overs
(2) (3)

3. Compare the steps that you designed for Alice with your classmates' steps. How are these steps similar to your steps?

NOTES

ACTIVITY 2.1

Scaling Up and Down on the Coordinate Plane

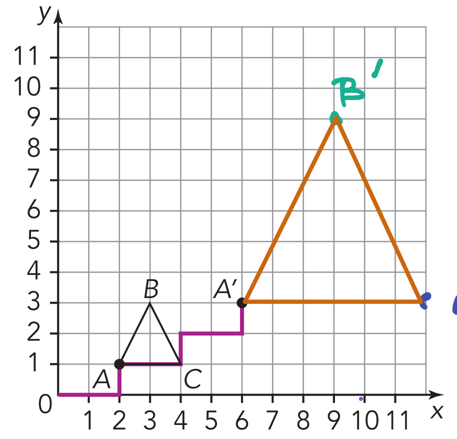
You know that a translation moves a point along a line. A sequence of repeated horizontal and/or vertical translations also moves a point along a line. You can use this fact to dilate figures.

WORKED EXAMPLE

Dilate $\triangle ABC$ by a scale factor of 3 using the origin as the center of dilation.

x3

Let's start by dilating Point A, which is located at (2, 1). In other words, Point A is translated from the origin 2 units right and 1 unit up.



To dilate point A by a scale factor of 3, translate Point A by three repeated sequences: 2 units right and 1 unit up from the origin.

1. Describe the repeated translations you can use to scale point *B* and point *C*. Then plot point *B*' and point *C*' on the coordinate plane in the worked example.

a. point *B* to point *B*′

up 3, over 3

b. point *C* to point *C*′

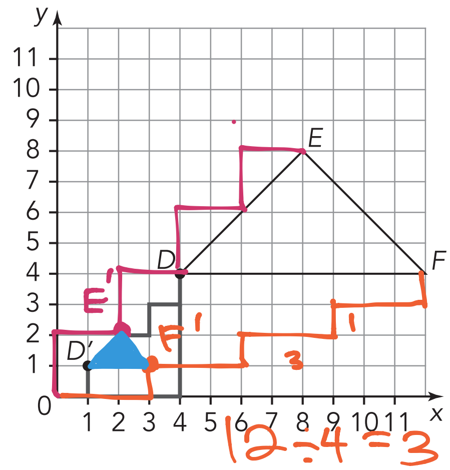
over 4, up 1

2. Draw $\triangle A'B'C'$ on the coordinate plane in the example. Is $\triangle ABC$ similar to $\triangle A'B'C'$? Explain your reasoning.

Yes, because they have same shape, diff size.

WORKED EXAMPLE

Dilate ΔDEF by a scale factor of $\frac{1}{4}$ using the origin as the center of dilation.

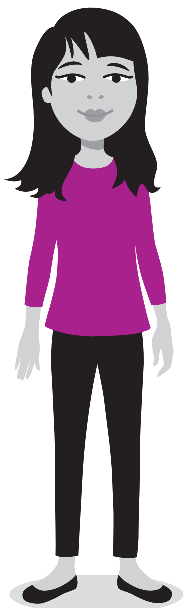


4 ÷ 4 = 1

Point D is translated from the origin 4 units right and 4 units up (4, 4). This is the same as four translations of 1 unit right and 1 unit up.

Therefore, scaling point D to (1, 1) represents a dilation by a scale factor of $\frac{1}{4}$.

"How do the side lengths and angles of the triangles compare?"



3. Determine the coordinates of points E' and F'. Explain how you determined your answers. Then, draw $\Delta D'E'F'$ on the coordinate plane in the example.

E′(2, 2) D′(1, 1)

F′(3, 1)

4. Is ΔDEF similar to $\Delta D'E'F'$? Explain your reasoning.

Yes, they have the same shape but diff size.

5. How does dilating a figure, using the origin as the center of dilation, affect the coordinates of the original figure? Make a conjecture using the examples in this activity.

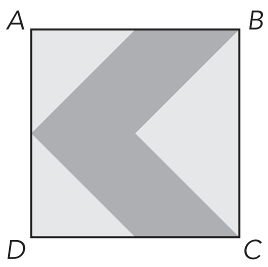
D (4, 4) D′(1, 1)
E (8, 8) E′(2, 2)
F (12, 4) F′(3, 1)

÷4

ACTIVITY
2.2

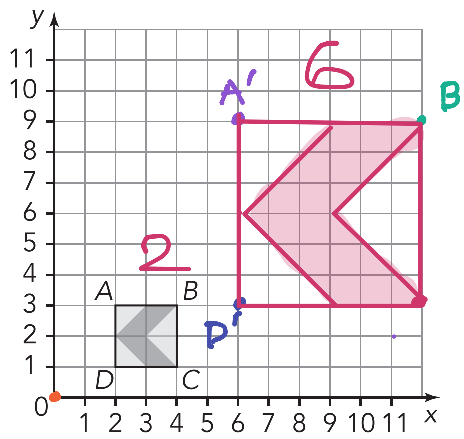
Using the Origin as the Center of Dilation

Road signs maintain a constant scale, regardless of whether they are on the road or in the drivers' manual. This sign indicates that the road is bending to the left.



1. Dilate the figure on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of 3 to form a new figure.

bigger



2. List the ordered pairs for the original figure and for the new figure. How are the values in the ordered pairs affected by the dilation?

A (2, 3)   A' (6, 9)
B (4, 3) ×3 B' (12, 9)
C (4, 1)   C' (12, 3)
D (2, 1)   D' (6, 3)

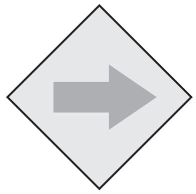
(3x, 3y)

3. Compare and contrast the corresponding angles and corresponding side lengths of the new figure and the original figure.

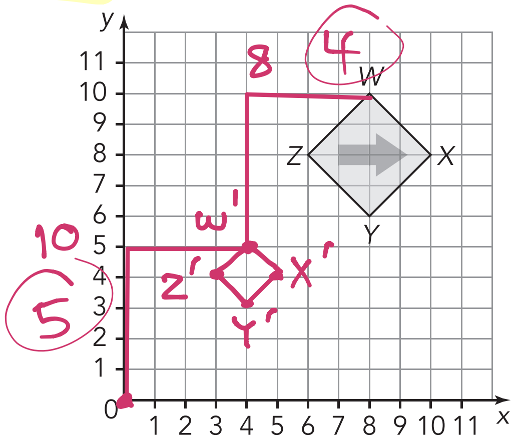
Corresponding angles are $\cong$.
Corresponding side length

$\frac{\text{new}}{\text{original}} = \frac{A'B'}{AB} = \frac{6}{2} = 3$

Let's consider a different road sign. This sign indicates that the road proceeds to the right.



4. Dilate the figure on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of $\frac{1}{2}$ to form a new figure.



5. List the ordered pairs for the original figure and for the new figure. How are the values in the ordered pairs affected by the dilation?

$\left(\frac{x}{2}, \frac{y}{2}\right)$

W (8, 10) $\div 2$ W' (4, 5)

X (10, 8) $\div 2$ X' (5, 4)

Y (8, 6) $\div 2$ Y' (4, 3)

Z (6, 8) $\div 2$ Z' (3, 4)

6. Compare and contrast the corresponding angles and corresponding side lengths of the original figure and the new figure.

corresponding angles are $\cong$

$\frac{\text{new}}{\text{original}} = \frac{1}{2}$

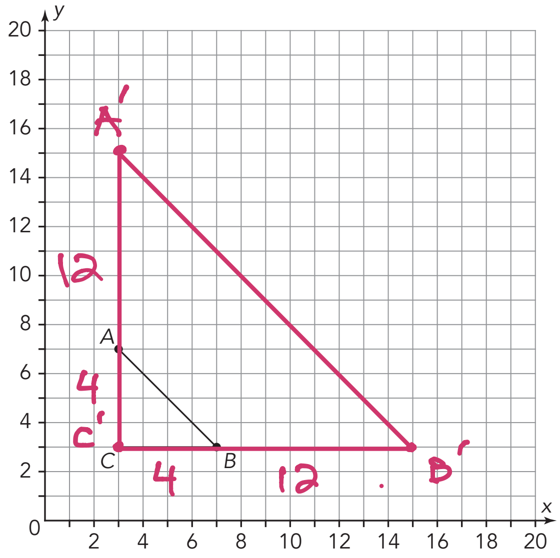
ACTIVITY 2.3

Using a Point on the Figure as a Center of Dilation

NOTES

You can use any point as the center of dilation. The center of dilation can be on the figure, inside the figure, or outside the figure.

1. Consider △*ABC*.



a. Dilate △*ABC* using point *C* as the center of dilation and a scale factor of 3 to form △*A′B′C′*. Explain how you determined the coordinates of the dilated figure.

- Find the distance from C to A and mult. it by 3 4×3=12

- Same for C to B

b. What are the coordinates of points *A′*, *B′* and *C′*?

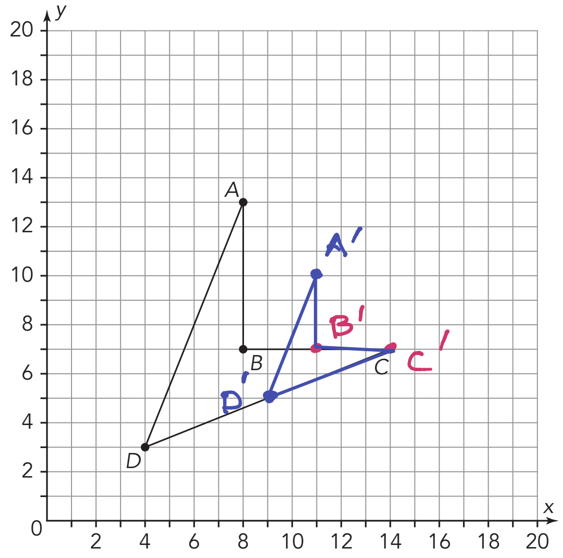
A (3, 7) A′ (3, 15)

B (7, 3) B′ (15, 3)

* C (3, 3) *C′ (3, 3)

can not mult by 3

2. Consider Quadrilateral *ABCD*.



a. Dilate Quadrilateral *ABCD* using point *C* as the center of dilation and a scale factor of $\frac{1}{2}$ to form Quadrilateral *A′B′C′D′*. Explain how you determined the coordinates of the dilated figure.

b. What are the coordinates of points *A′*, *B′*, *C′*, and *D′*?

A (8, 13)   A' (11, 10)
B (8, 7)   B' (11, 7)
* C (14, 7)   * C' (14, 7)
D (4, 3)   D' (9, 5)

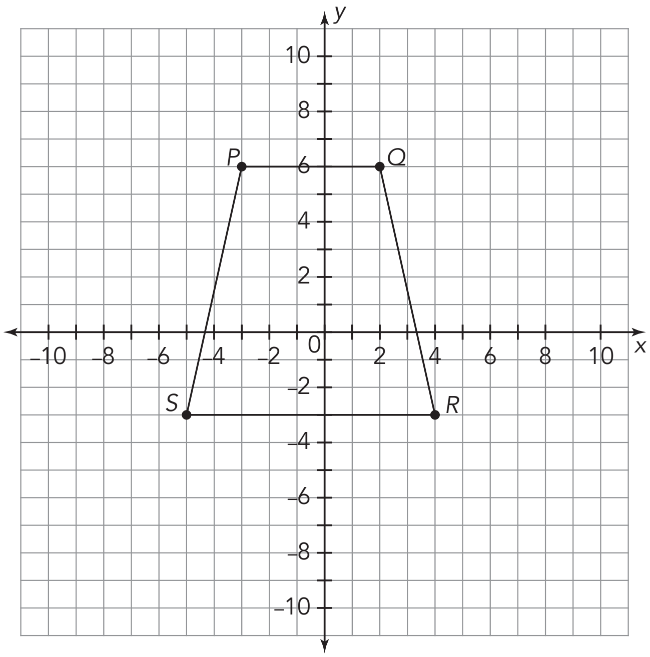
c. How are the coordinates of a figure affected by a dilation that is not centered at the origin?

cannot simply divide by 2 because the center of dilation is not at the origin.

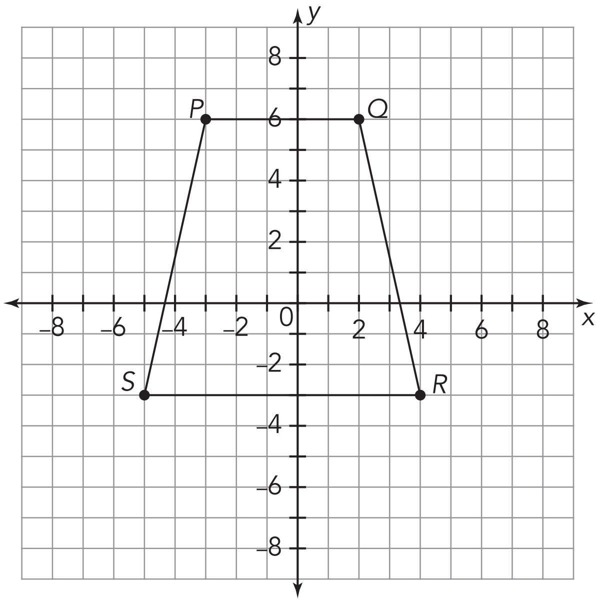
ACTIVITY 2.4	Using a Point Inside or Outside the Figure as a Center of Dilation

In this activity, you will explore different center points for dilation to understand how the coordinates of a figure are affected by dilations.

1. Dilate Figure *PQRS* by a scale factor of $\frac{3}{2}$ using the point (4, 6) as the center of dilation. Determine the coordinates of Figure *P′Q′R′S′* and draw the approximate dilation on the coordinate plane.



2. **Dilate Figure *PQRS* by a scale factor of $\frac{2}{3}$ using the point (–2, 0) as the center of dilation. Determine the coordinates of Figure *P′Q′R′S′* and draw the approximate dilation on the coordinate plane.**



3. **How are the coordinates of a figure affected by a dilation that is not centered at the origin? How do you think you can modify your original conjecture?**

$$A(\overset{\times 2}{1}, \overset{\times 2}{3}) \qquad A'(2, 6)$$
$$B(\overset{\times 2}{-1}, \overset{\times 2}{5}) \xrightarrow[\text{about the origin}]{\text{dilate with scale factor 2}} B'(-2, 10)$$
$$C(\overset{\times 2}{-3}, \overset{\times 2}{7}) \qquad C'(-6, 14)$$

* * *

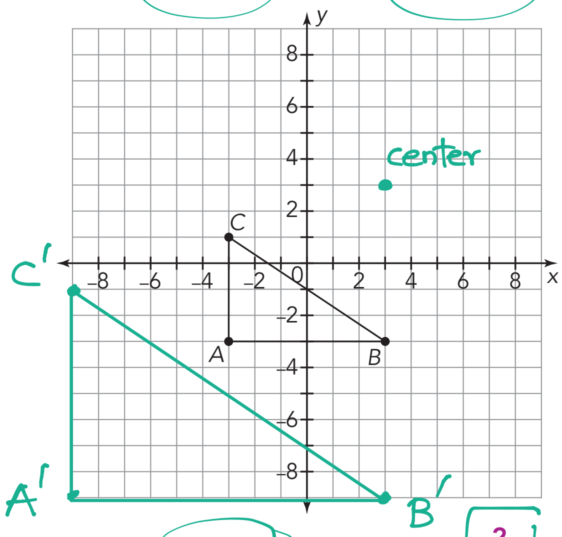
If the dilation of a figure is centered at the origin, you can multiply the coordinates of the points of the original figure by the scale factor to determine the coordinates of the new figure.

To determine the dilation of a figure not centered at the origin, you can follow these steps:

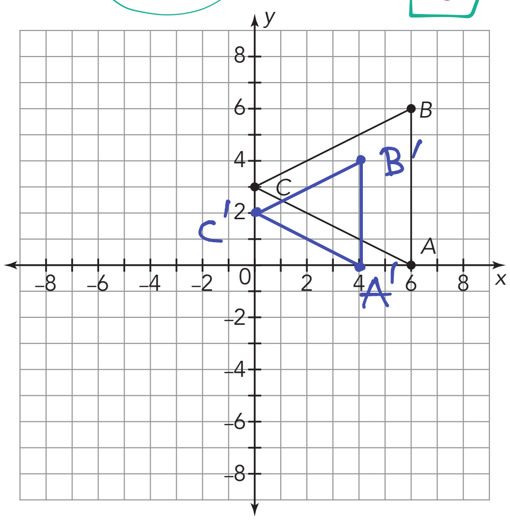
- Subtract the *x*- and *y*-coordinates of the center from the *x*- and *y*-coordinates of each point.
- Multiply the new coordinates of each point by the scale factor.
- Add the *x*- and y-coordinates of the center to the new *x*- and *y*-coordinates of each point.

4. Determine the dilation of each triangle using the information given. Verify your answer on the coordinate plane.

a. Center: (3, 3) Scale factor: 2



b. Center: origin Scale factor: $\boxed{\frac{2}{3}}$



A (6, 0) A'(4, 0)

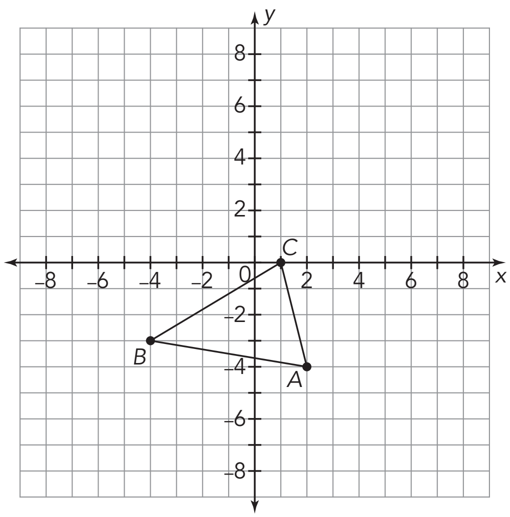
B (6, 6) $\xrightarrow{\times \frac{2}{3}}$ B'(4, 4)

C (0, 3) C'(0, 2)

$$\frac{6}{1} \times \frac{2}{3} = \frac{12}{3} = 4$$

$$\frac{3}{1} \times \frac{2}{3} = \frac{6}{3} = 2$$

c. Center: (1, –3) Scale factor: 2



NOTES

TALK the TALK

Location, Location, Location

Answer each question to summarize what you know about dilating figures on the coordinate plane. Use your answers to plan a presentation for your classmates that demonstrates what you learned in this lesson.

1. What strategies can you use to determine if two figures are similar when they are:

a. located on a coordinate plane?

b. not located on a coordinate plane?

2. How does the location of the center of dilation affect the coordinates of the dilated figure?

3. Describe how you can determine whether two figures on the coordinate plane are similar using just their coordinates and the center of dilation.

Assignment

Write

In your own words, explain how to dilate a figure on the coordinate plane using repeated translations. Use examples with scale factors less than and greater than 1 to illustrate your explanation.

Remember

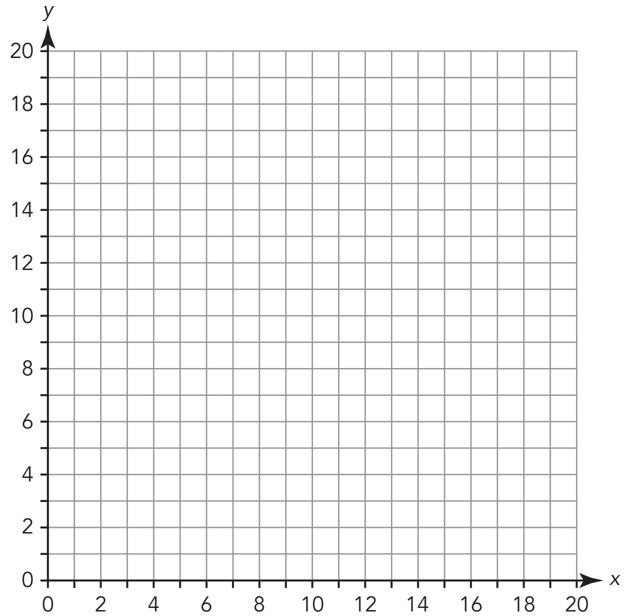
If the dilation of a figure is centered at the origin, you can multiply the coordinates of the points of the original figure by the scale factor to determine the coordinates of the new figure.

To determine the dilation of a figure not centered at the origin, you can follow these steps:

- Subtract the *x*- and *y*-coordinates of the center from the *x*- and *y*-coordinates of each point.
- Multiply the new coordinates of each point by the scale factor.
- Add the *x*- and *y*-coordinates of the center to the new *x*- and *y*-coordinates of each point.

Practice

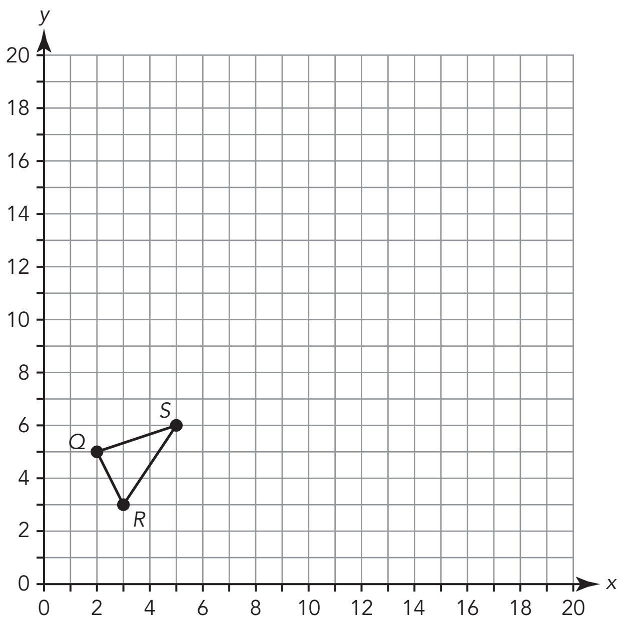
1. Graph Triangle *XYZ* with the coordinates *X* (2, 17), *Y* (17, 17), and *Z* (17, 8).



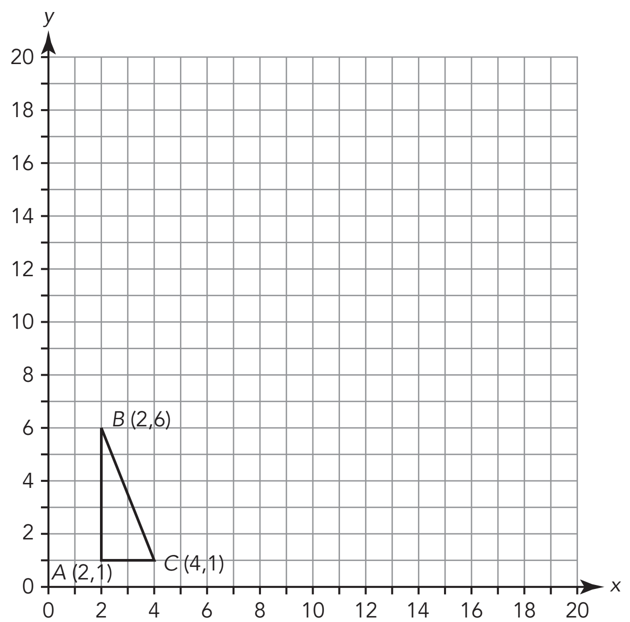
a. Reduce Triangle *XYZ* on the coordinate plane using the point *Y* as the center of dilation and a scale factor of $\frac{1}{3}$ to form Triangle *X′YZ′*.

b. What are the coordinates of points *X′* and *Z′*?

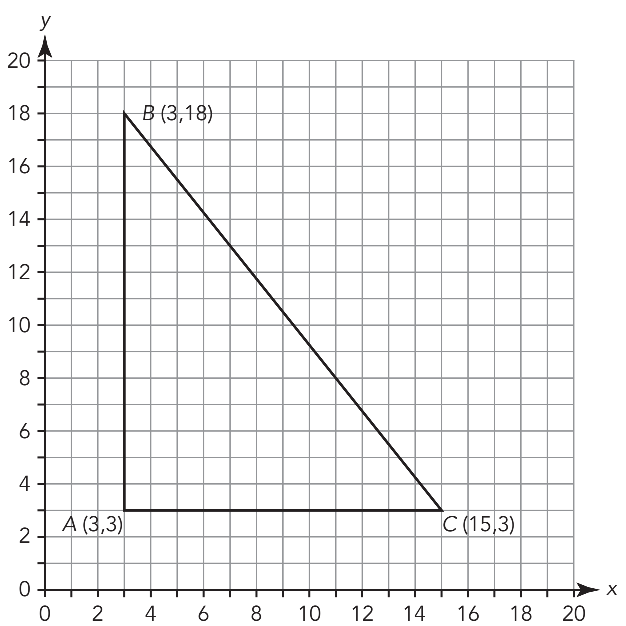
2. Dilate Triangle *QRS* on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of 3 to form Triangle *Q′R′S′*. Label the coordinates of points *Q′*, *R′*, and *S′*.



3. Dilate Triangle *ABC* on the coordinate plane using point *A* (2, 1) as the center of dilation and a scale factor of 3.

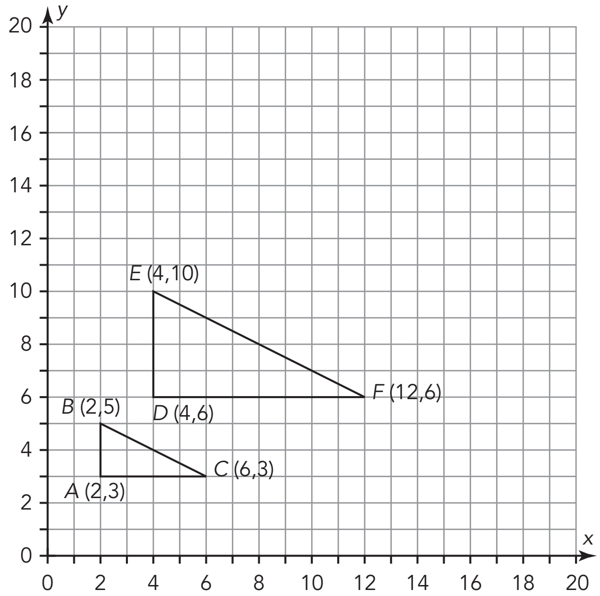


4. Dilate Triangle *ABC* on the coordinate plane using point *A* (3, 3) as the center of dilation and a scale factor of $\frac{1}{3}$.

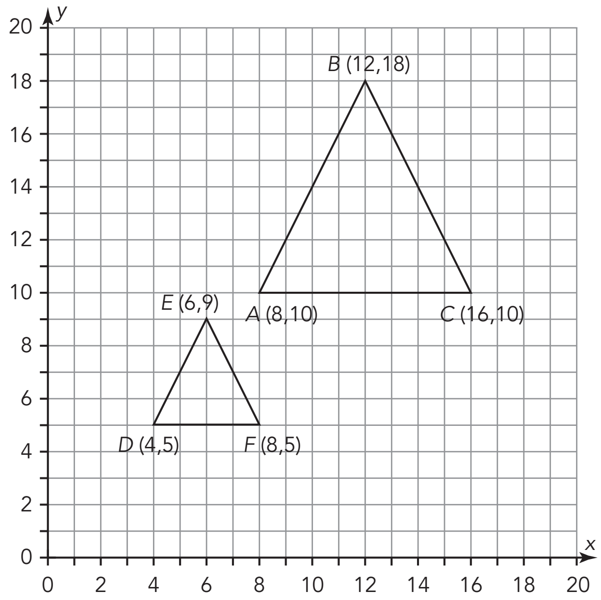


5. Verify that each pair of triangles is similar.

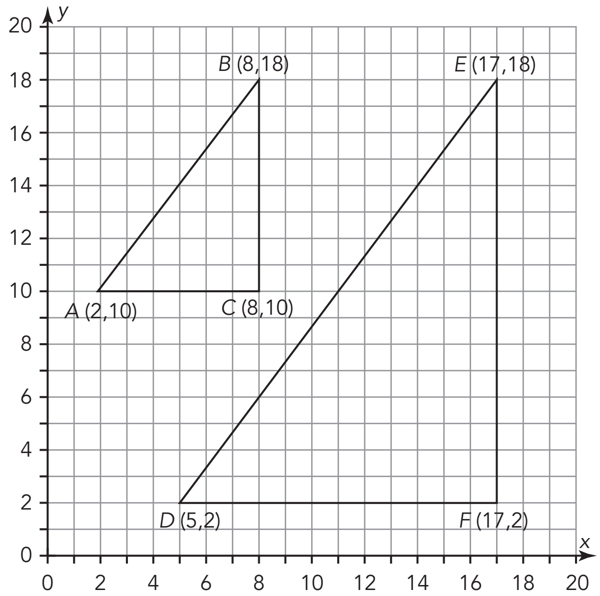
a.



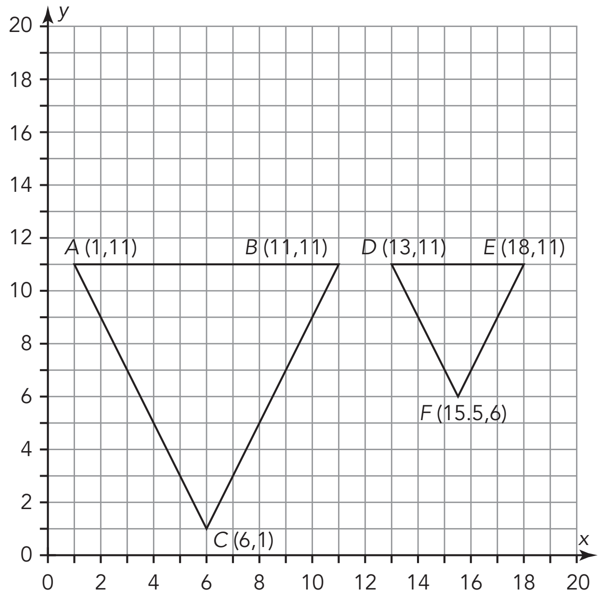
b.



c.



d.

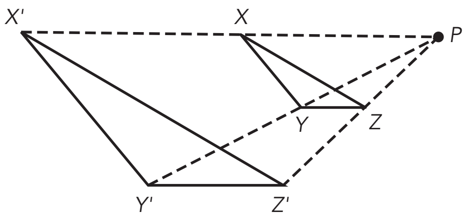


Stretch

Square *ABCD* has coordinates *A* (4, 4), *B* (8, 4), *C* (8, 0), and *D* (4, 0). A dilation of Square *ABCD* has coordinates *A′* (0, 0), *B′* (2, 0), *C′* (2, −2), and *D′* (0, −2). What is the center of dilation?

Review

1. Triangle XYZ has been enlarged with P as the center of dilation to form Triangle $X'Y'Z'$. Identify the equivalent ratios that are equal to the scale factor.



2. A triangle is dilated with center of dilation at point U. Point E is a vertex of the triangle, and point E' is the corresponding vertex of the image. If $UE = 2$ centimeters and $UE' = 10$ centimeters, what is the scale factor?
3. The coordinates of Quadrilateral $ABCD$ are A (−6, 2), B (−5, 3), C (7, 3), and D (0, −4). What are the coordinates of the image if the quadrilateral is translated 4 units right and 3 units down?
4. The coordinates of ΔJKL are J (0, 1), K (6, 0), and L (−6, 0). What are the coordinates of the image if the triangle is translated 8 units left?
5. Write two unit rates for each situation.
   a. Julie can deliver $\frac{1}{4}$ of the newspapers in $\frac{1}{2}$ hour.
   b. It took the author $\frac{3}{4}$ of the year to write $\frac{1}{4}$ of the book.