

The Road Less Traveled

2

Systems of Linear Equations

WARM UP

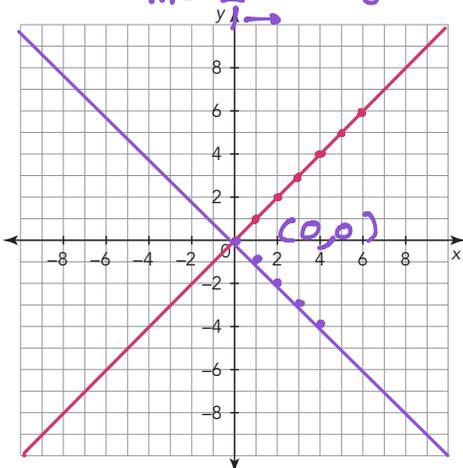
1. Graph the equations on the coordinate plane. $y = mx + b$

$$y = 1x$$

$$m = 1 \uparrow \quad b = 0$$

$$y = -x$$

$$m = -1 \downarrow \quad b = 0$$



2. What are the coordinates of the point of intersection?
3. Interpret the meaning of the point of intersection.

LEARNING GOALS

- Write a system of equations to represent a problem context.
- Analyze and solve a system of two simultaneous linear equations in two variables graphically.
- Interpret the solution to a system of equations in terms of a problem situation.
- Use slope and y-intercept to determine whether two linear equations have one solution, no solutions, or infinite solutions.

KEY TERMS

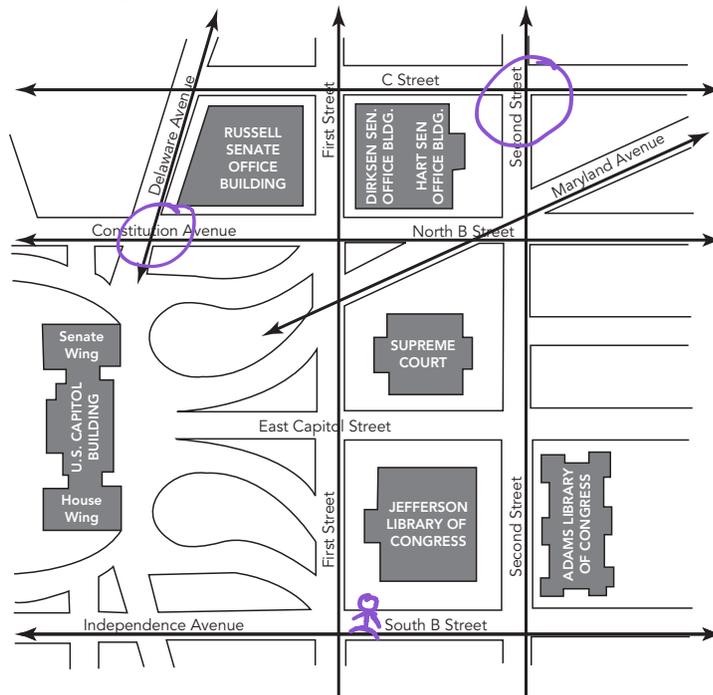
- system of linear equations
- solution of a linear system
- consistent system
- inconsistent system

You have graphed linear equations on a coordinate plane. How can you interpret two linear equations together as a system?

Getting Started

According to the Map

Many of the diagonal roads in Washington, DC, are named after US states. Except for California and Ohio, every state provides the name for an avenue. California is a street, and Ohio is a drive. There is also a Puerto Rico Avenue.



1. Answer each question and explain your reasoning according to the map shown.

a. Would it be possible to meet a friend at the intersection of First Street and Second Street?

No, they are parallel!

b. Would it be possible to meet a friend at the intersection of Delaware Avenue and Constitution Avenue?

Yes

c. Would it be possible to meet a friend at the intersection of C Street and Second Street?

Yes

2. How many places could you be if you are at the intersection of Independence Avenue and South B Street?

Infinitely many

ACTIVITY
2.1

Representing a Problem Situation with a System of Equations



Colleen and Jimmy have part-time jobs after school. Both have decided that they want to see how much money they can save in one semester by placing part of their earnings each week into a savings account. Colleen currently has \$120 in her account and plans to save \$18 each week. Jimmy currently has \$64 in his savings account and plans to save \$25 each week.

- Write an equation for Colleen and for Jimmy that represents the total amount of money, in dollars, in each of their savings accounts, y , in terms of the number of weeks, x , that they place money in their respective accounts.

$x = \# \text{ of weeks}$
 $y = \text{total saving}$

Colleen
 $y = 18x + 120$

Jimmy
 $y = 25x + 64$

- How much money will each person have in his or her savings account after five weeks? $5 = x$ ☺

Colleen	$18(5) + 120$	Jimmy	$25(5) + 64$
	$90 + 120$		$125 + 64$
	$\$210$		$\$189$

← saves more

- Which person will have more money in his or her savings account after five weeks?

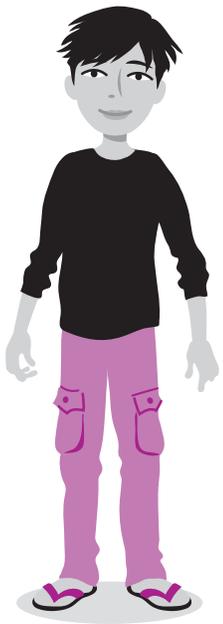
→ Colleen saves more at the moment

- How much money will each person have in his or her savings account after 18 weeks (the amount of time in one semester)?

Colleen → $y = 18(18) + 120$
 $y = 324 + 120$
 $\$444$

Jimmy → $y = 25(18) + 64$
 $y = 450 + 64$
 $\$514$

The slope of a line is its rate of change.

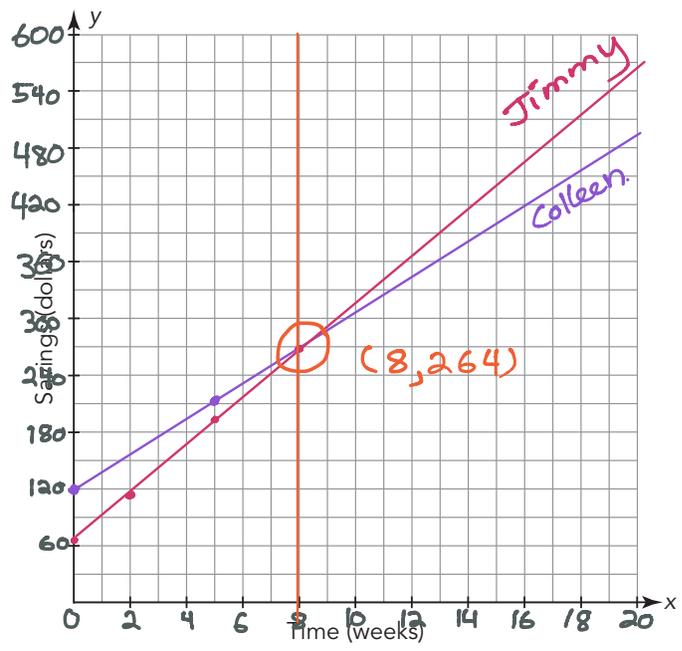


5. Which person will have more money in his or her savings account at the end of the semester?

Jimmy

6. Create a graph of each equation on the coordinate plane shown. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval



# of weeks	Colleen	Jimmy
X	$18x + 120$	$25x + 64$
0	120	64
2	156	114
5	210	189
8	264	264
12	336	364
18	444	514

7. Determine the number of weeks after which Colleen and Jimmy will have the same amount of money in their savings accounts.

8 weeks

8. Verify your solution to Question 7 algebraically.

9. Interpret the meaning of the slope of each graph in this problem situation.

Colleen: $m = 18 \rightarrow$ she saves \$18/week
 Jimmy: $m = 25 \rightarrow$ he saves \$25/week.

$$\begin{array}{r}
 18x + 120 = 25x + 64 \\
 -18x \quad -18x \\
 \hline
 120 = 7x + 64 \\
 -64 \quad -64 \\
 \hline
 56 = 7x \\
 8 = x
 \end{array}$$

10. Which person is saving more money per week?

Jimmy

11. How can you tell who is saving more money each week by analyzing the graph?

Steeper line.

12. Interpret the meaning of the y-intercept of each graph in this problem situation.

Colleen: \$120 → She already had \$120
Jimmy: \$64 → He " " " \$64

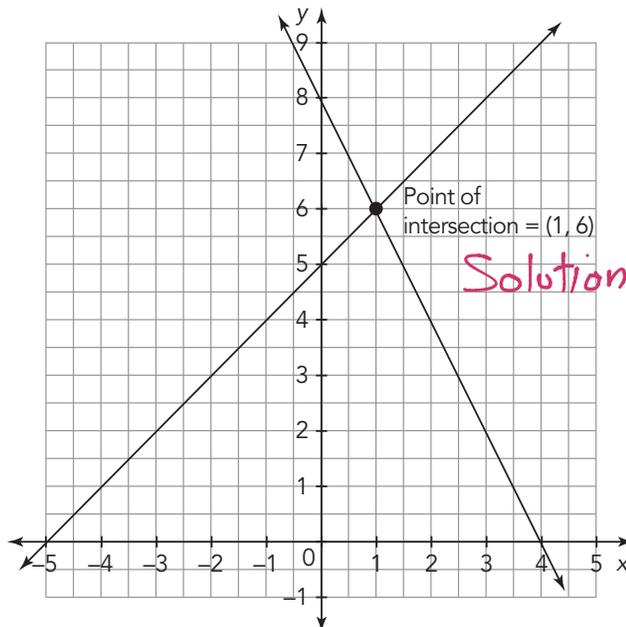
When two or more linear equations define a relationship between quantities, they form a **system of linear equations**. The **solution of a linear system** is an ordered pair (x, y) that is a solution to both equations in the system. Graphically, the solution is the point of intersection, the point at which two or more lines cross.

WORKED EXAMPLE

A system of linear equations is written with a brace as shown:

$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

You can determine the solution to this system by graphing the equations. The point of intersection is the solution to the system.



ACTIVITY
2.2

Systems with One Solution and No Solutions



Eric also has a part-time job after school working at the same place as Jimmy. He heard about the money that Colleen and Jimmy were saving and decided that he wanted to save money, also. Eric has \$25 in his savings account and will save the same amount as Jimmy, \$25 per week.

$$y = 25x + 64$$

1. Write an equation that represents the total amount of money in Eric's savings account, y , in terms of the number of weeks, x , that he places money in his savings account.

Eric:

$$y = \underline{25x} + \underline{25}$$

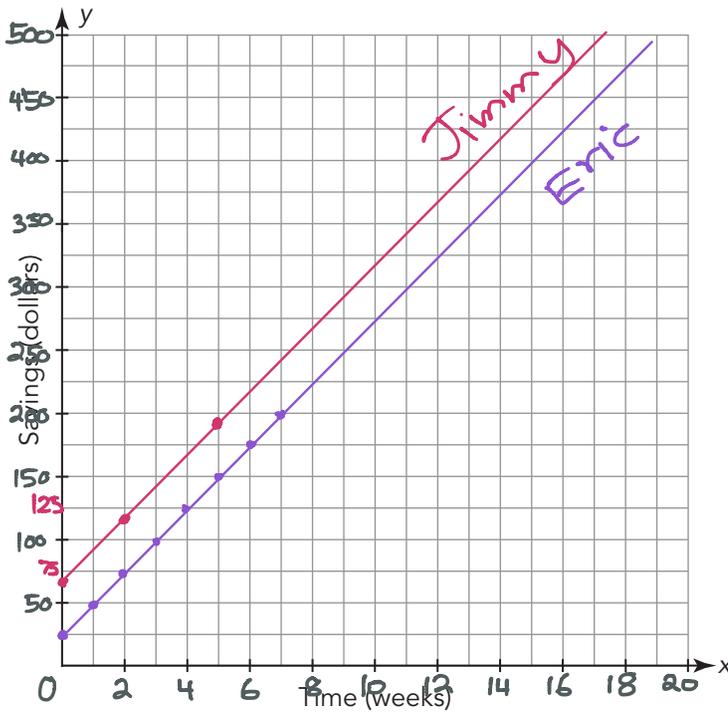
2. Write a linear system that shows the total amount of money that will be saved by Eric and Jimmy.

Jimmy : $y = 25x + 64$

Eric : $y = 25x + 25$

3. Create a graph of the linear system on the coordinate plane shown. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval



Jimmy	x	y
	0	64
	2	114
	5	189
	8	264
	12	364

Eric: $y = 25x + 25$
 $b = 25$
 $m = \frac{25}{1} \rightarrow$

4. What does the slope of each graph represent in this problem situation? 25

How much each saved per week.

5. What is the same for both Eric and Jimmy?

the slope $\rightarrow m$

6. What is different for Eric and Jimmy?

the initial amount $\rightarrow b$

7. What is the point of intersection for this system of equations?
 Explain your reasoning in terms of the graph.

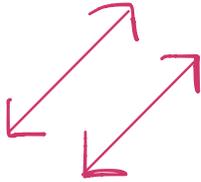
None

The lines you graphed in Question 3 are parallel lines. Remember that two lines are parallel if they lie in the same plane and do not intersect.

8. What do you know about the slopes of parallel lines?

They are the same.

9. Does the linear system of equations for Eric and Jimmy have a solution? Explain your reasoning in terms of the graph.



No solution \rightarrow no intersection point.

10. Will Eric and Jimmy ever have the same amount of money in their savings accounts?

Never.

Eric's sister Trish was able to save \$475 working part-time during the first semester of school. She recently quit her part-time job to play on the high school's softball team. She is hoping to get a college scholarship to play softball and wants to devote her time to achieving her goal. She will withdraw \$25 each week from her savings account for spending money while she is not working.

11. Write an equation that gives the total amount of money in Trish's savings account, y , in terms of the number of weeks, x , that she withdraws money out of her savings account.

$$y = 475 - 25x$$

$$y = -25x + 475$$

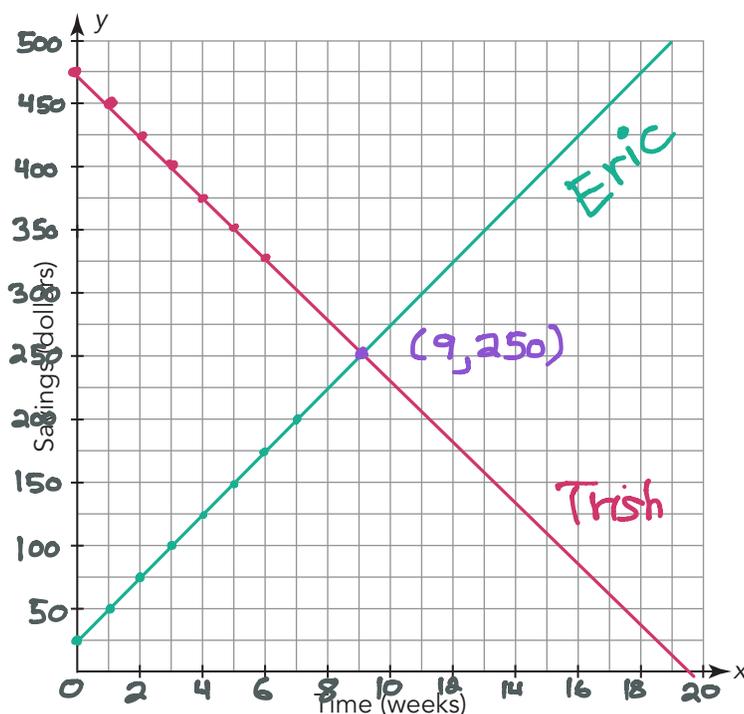
12. Write a system of equations that represents the amount of money that Trish and Eric will have in their respective savings accounts.

$$\text{Eric: } y = 25x + 25$$

$$\text{Trish: } y = -25x + 475$$

13. Create a graph of the linear system on the coordinate plane shown. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval



Eric:

$$y = 25x + 25$$

$$m = \frac{25 \uparrow}{1 \rightarrow}$$

$$b = 25$$

Trish:

$$y = -25x + 475$$

$$m = \frac{-25 \downarrow}{1 \rightarrow}$$

$$b = 475$$

14. What does the point of intersection of the lines represent?

Where ⁽⁹⁾ both have the same amount of money (250)

15. Compare the slopes of the lines.

One is positive

One is negative

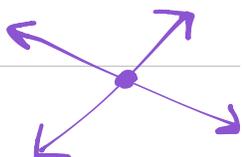
16. According to the graph, approximately when will Trish and Eric have the same amount of money in their savings accounts? How much will they each have?

The 9th week of saving, \$250



One solution

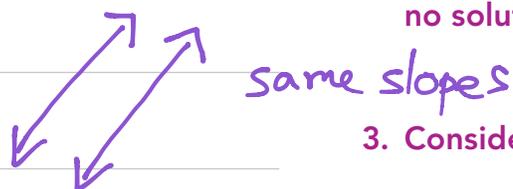
You have worked with systems of linear equations that have one solution and no solutions.



1. Describe the graphs in a system of linear equations that has one solution.

No solution

2. Describe the graphs in a system of linear equations that has no solution.

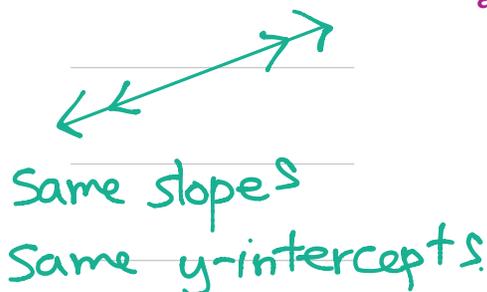


3. Consider the system of equations:

In finitely many solutions

$$\begin{cases} y = 3x + 6 \\ y = 3(x + 2) = 3x + 6 \end{cases}$$

a. Complete the table of values for this linear system.



x	$y = 3x + 6$	$y = 3(x + 2)$
-2	$3(-2) + 6 = 0$	$3(-2 + 2) = 0$
0	$3(0) + 6 = 6$	$3(0 + 2) = 6$
2		
4		
8		
13		
20		

b. Describe the equations that make up this system. What can you conclude about the number of solutions to this type of linear equation?

2. Using only the equations, determine whether each system has one solution, no solutions, or infinite solutions. Explain your reasoning.

a. $y = \frac{4}{5}x - 3$ and $y = -\frac{5}{4}x + 6$

b. $y = \frac{2}{3}x + 7$ and $y = \frac{1}{6}(4x + 42)$

c. $y = -2.5x + 12$ and $y = 6 - 2.5x$

d. $y = 5x$ and $y = \frac{1}{5}x$

A system of equations may have one unique solution, infinite solutions, or no solutions. Systems that have one or infinite solutions are called **consistent systems**. Systems that have no solution are called **inconsistent systems**.

3. Complete the table.

	Consistent Systems		Inconsistent Systems
	One Unique Solution	Infinite Solutions	No Solutions
Compare the slopes.			
Compare the y-intercepts.			
Describe the lines.			

Assignment

Write

Complete each sentence by writing the correct term or phrase from the lesson.

1. A(n) _____ is formed when the equations or graphs of two or more linear equations define a relationship between quantities.
2. A(n) _____ is an ordered pair (x, y) that is the point of intersection, the point at which two or more lines cross.
3. A(n) _____ has one or infinite solutions.
4. A(n) _____ has no solution.

Remember

- A system of equations whose graphs intersect at just one point is a system with one solution.
- A system of equations that has parallel line graphs is a system with no solutions.
- A system of equations that has identical graphs is a system with infinite solutions.

Practice

Aiko works in the fish department of a pet store. She is asked to drain, clean, and refill two reef tanks. The first tank holds 175 gallons of water, and the second tank holds 200 gallons of water. The hoses that she uses drain the tanks at a rate of 25 gallons of water per hour.

1. Write an equation for each tank that represents the total amount of water in gallons in the tank, y , in terms of the number of hours, x , that the tanks are draining.
2. How much water is in each tank after 3 hours?
3. Write your equations in the first row of the table. Then, complete the table of values for the linear system.

Number of Hours	First Tank	Second Tank
x		
0		
1		
2		
3		
4		
5		
6		
7		

4. Create a graph of both equations.
5. Interpret the meaning of the slope of each line in this problem situation.
6. What is the same for both tanks?
7. What is different for the two tanks?
8. What is the point of intersection for this system of equations? Explain your reasoning in terms of the graph.
9. When will both tanks have the same amount of water?
10. While Aiko is draining both tanks, she is also filling a 250-gallon tank. The water fills at a rate of 25 gallons per hour. Write an equation that gives the total amount of water in gallons in the third tank, y , in terms of the number of hours, x , that the tank is filling.

Stretch

A system with an equation that has an exponent of 2 can have more than one solution. How many solutions does the system $y = x$ and $y = x^2 - 2$ have? What are the solutions?

Review

1. Billy is selling lemonade for \$1 per cup. It costs him 50 cents per cup to make the lemonade. He also has to spend an additional \$10 for supplies such as ice, cups, and plastic shakers.
 - a. Write a system of equations to represent this situation.
 - b. What does the point of intersection represent in this situation?
2. Determine whether the equations have one solution, no solutions, or infinite solutions.
 - a. $1.5x + 6.5 = \frac{3}{2}x + \frac{13}{2}$
 - b. $-\frac{1}{5}x - 12 = -0.2x - \frac{24}{2}$
3. Solve each equation.
 - a. $4(x + 5) = 6(x + 4)$
 - b. $-3(p - 4) = -2p + 1$