Dining, Dancing, Driving

Linear Relationships in Contexts

3

WARM UP

The lunch special at the pizza shop is two slices of pizza for $5.00.

1. Express the cost of the pizza as a unit rate.

2. Create a table to represent this context.

Number of Slices of Pizza	Cost (dollars)

3. Write an equation to represent this situation. Define your variables.

LEARNING GOALS

- Determine the slope from a context.
- Connect the rate of change represented in a context to the rate of change in other representations.
- Interpret the rate of change of a linear relationship in terms of the situation it models.
- Generate the values of two coordinate pairs from information given in context.
- Determine the independent and dependent quantities from contexts.

You have analyzed linear relationships in graphs and tables. How can you determine rates of change from word problems alone?
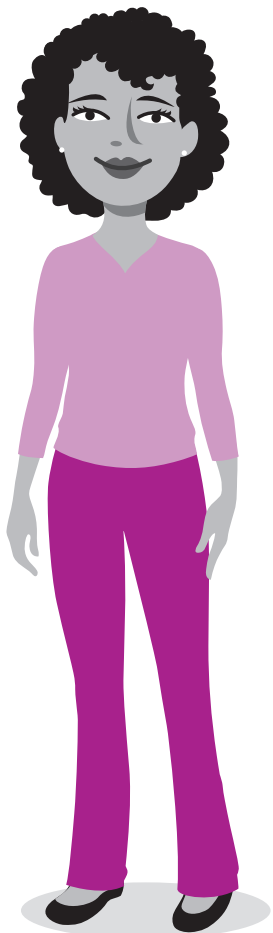
Dependent on Your Point of View

Identify the dependent quantity and the independent quantity in each problem situation.

(handwritten above "dependent quantity": y) (handwritten above "independent quantity": x)

> Remember, the dependent quantity is the variable whose value is determined by an independent quantity.

1. Terrence is purchasing canned vegetables at his local grocery store to donate to the local food pantry. Each can costs $0.59.

   independent (x) → the # of cans purchased
   dependent (y) → the total cost.

2. The amount of electricity used by a light changes as the knob on the dimmer switch is turned.

   independent (x) → the position of the knob
   dependent (y) → the brightness

3. Stephanie is selling Girl Scout cookies to raise money for her local troop. For each box of cookies she sells, the troop receives $2.00.

   indep (x) → the # of boxes sold
   dep (y) → the total made

4. How would each problem situation change if you switched the independent and dependent quantities? Would each problem still make sense?

Choosing Independent and Dependent Quantities

You can choose different independent and dependent quantities to model the same information, depending on what you want to know. Once you have determined the independent and dependent quantities, you need just two points to determine the slope, or unit rate.

Josh took a road trip with his family to visit Yosemite National Park in California. Some information about their trip is shown in the table.

Total Miles	Total Cost for Gas ($)	Total Gallons
2600 *y*	~~200~~	80 ✗

1. After they arrived, Josh was curious about **how many miles per gallon** their car got on the trip.

   $$\frac{miles}{gallons} \leftarrow \frac{dependent}{independent} \quad \frac{y}{x}$$

   a. Given this question, what are the independent and dependent quantities?

   indep (x) → the # of gallons used
   dep (y) → the # of miles drove

   b. Write the ordered pairs of two points you can use to answer the question. Explain what each of your ordered pairs means in terms of the situation.

   $$(\underset{x_1}{80}, \underset{y_1}{2600}) \quad (\underset{x_2}{0}, \underset{y_2}{0})$$

   c. Determine the rate. Explain what this means in terms of the problem situation.

   $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2600}{0 - 80} = \frac{-2600}{-80} = \frac{2600}{80} = \frac{32.5 \text{ miles}}{1 \text{ gallon}}$$

   d. How many miles per gallon did their car get on the road trip?

   ✳ The car could go 32.5 miles per one gallon

2. The family wants to know about how many gallons of gas on average they used for each mile of the trip.

   a. Given this question, what are the independent and dependent quantities?

   b. Write the ordered pairs of two points you can use to answer the question. Explain what each of your ordered pairs means in terms of the situation.

   c. Determine the rate. Round to the nearest hundredth. Explain what this means in terms of the problem situation.

   d. What was the family's average gallons per mile for the trip?

3. If the family had flown, they would have traveled 2100 miles and spent $3250 for tickets alone. Compare the costs per mile for flying and driving. Determine the independent and dependent quantities and rates for each relationship. Show your work.

For each context, complete each task.

- Identify the independent and dependent quantities.
- Write the ordered pairs of two points you can use to answer the question. Explain what each of your ordered pairs means in terms of the situation.
- Then, determine the rate described.

1. Bella's Pizza Shop charges $4.50 for a small pizza, $7.00 for a medium pizza, and $9.00 for a large pizza. Toppings cost extra, depending on the size of the pizza ordered. Bruce ordered a large pizza with three toppings that cost a total of $12.60. What is the cost per number of toppings for a large pizza?

$$m = \frac{\text{the cost}}{\text{the \# of toppings}} \xleftarrow{} \text{dep}$$
$$\xleftarrow{} \text{indep}$$

$$(3, 12.60) \quad (0, 9)$$
$$\quad x_1 \quad y_1 \qquad x_2 \quad y_1$$

$$m = \frac{9-12.6}{0-3} = \frac{-3.6}{-3} = \$1.20$$
$$\frac{}{1 \text{ topping}}$$

It costs $1.20 per topping for a large pizza.

2. A maintenance crew is paving a road in 7-hour shifts. After 10 shifts, 1.25 miles of road have been paved. After 45 shifts, 5.625 miles of road have been paved. At what rate is the maintenance crew paving the road in miles per shift?

# of miles (y)
# of shifts (x)

$$(10, 1.25) \quad (45, 5.625)$$
$$\quad x_1 \quad y_1 \qquad x_2 \quad y_2$$

$$m = \frac{5.625 - 1.25}{45 - 10} = \frac{4.375}{35} = \frac{0.125 \text{ mi}}{1 \text{ shift}}$$

The crew can pave 0.125 mi per one shift.

3. Melanie is baking breakfast rolls for a band camp fundraiser. She bakes 15 dozen breakfast rolls in 3 hours. After 8 hours, she has baked 40 dozen breakfast rolls. At what rate does Melanie bake breakfast rolls each hour?

# of dozens baked (y)
# of hours spent (x)

$$(3, 15) \quad (8, 40)$$
$$\quad x_1 \quad y_1 \qquad x_2 \quad y_2$$

$$m = \frac{40-15}{8-3} = \frac{25}{5} = \frac{5 \text{ dozens}}{1 \text{ hour}}$$

Melanie can bake 5 dozens

in one hour.

4. Aleesa's dog, Bull, has been put on a diet by his veterinarian. He weighs 149 pounds after 8 weeks on his diet. By Week 13, he weighs 134 pounds. What is his average weight loss per week?

$(\underset{x_1}{8} , \underset{y_1}{149})$ $(\underset{x_2}{13} , \underset{y_2}{134})$

$\dfrac{\text{weight loss } (y)}{\# \text{ of weeks } (x)}$ $m = \dfrac{134 - 149}{13 - 8} = \dfrac{-15}{5} = \dfrac{-3}{1}$

The dog loses 3 lbs per one week.

Solve each problem.

5. Kathy is working after school to finish assembling the 82 favors needed for the school dance. When she starts at 3:15 PM, she counts the 67 favors that are already assembled. She works until 4:30 PM to finish the job.

a. How many favors can Kathy assemble each minute?

b. How many minutes does it take Kathy to assemble one favor?

c. Which rate is more meaningful in this situation? Explain your reasoning.

6. Eddie rented a moving van to travel across the country. The odometer registered 34,567 miles after he drove for 4 hours. After 7 hours of driving, the odometer read 34,741 miles. What was Eddie's driving rate in miles per hour?

7. Julie used her gift card for the local coffee shop to buy iced teas for herself and five friends. After she and one friend placed their orders, the balance on Julie's gift card was $14.85. After all six members of the group got their iced teas, she had a balance of $3.97 on her gift card. Determine the cost for one glass of iced tea.

TALK the TALK

And Stamps and Tickets

1. A book of 20 postage stamps costs $8.80. What is the cost of one postage stamp?

2. Ticket sales for a local concert totaled $101,244 yesterday. After the ticket window closed today, the cashiers counted 968 tickets sold with a two-day total of $143,836. What is the cost of one concert ticket?

y [underline] *x*

y = the total sales.

x = # of tickets sold

Number of Tickets Sold Today	Total Amount of Sales (dollars)
x_1 968	y_1 143,836
x_2 0	y_2 101244

$$m = \frac{101244 - 143836}{0 - 968} = \frac{-42592}{-968} = \$44 \overline{1 \text{ ticket.}}$$

3. List two similarities between Questions 1 and 2.

4. List two differences between Questions 1 and 2.

Assignment

Write

Describe how to use the independent and dependent quantities in a word problem to determine the rate of change, or slope.

Remember

Two ordered pairs are needed to determine a unit rate given a real-world problem situation.

Practice

1. Lashawna is making jewelry to sell at a craft fair. On Monday, she makes 12 bracelets. On Tuesday, she works an additional 2.5 hours and has a total of 22 bracelets. Determine the time it takes Lashawna to make one bracelet.

2. Nina and her friends are going to the downtown rib festival. The festival organizers expect 10,000 people to attend the four-day festival. At the end of the festival, the organizers say that they have exceeded their expected attendance by 2000 people. Determine the average number of people that attended the festival per day.

3. Aiko spends 2.5 hours baking croissants for a community center bake sale. She bakes the 90 croissants in 5 batches. Determine the number of batches Aiko baked per hour.

4. Nelson is selling his photographs at an art festival. The festival is open for 6 hours each day for 3 days. At the conclusion of the festival, Nelson has sold 54 photographs. Determine the number of photographs Nelson sold per hour.

5. Clayton wants to purchase tickets for the rides at a carnival. He can choose to purchase tickets individually, or he can purchase a ticket package. The package includes 25 tickets for $18.75. Determine the cost per ticket if he purchases the package.

6. Tameca is planning a hiking trip. The trail she would like to follow is 7.5 miles long. She plans to start her hike at 10:00 am. She hopes to reach the end of the trail at 3:00 pm. Determine the number of miles per hour that Tameca plans to hike.

Stretch

Create a situation that can be represented by a linear relationship whose unit rate value doesn't change when you switch the independent and dependent quantities.

Review

Determine whether the relationships represented in the tables are linear. If so, calculate the rate of change.

1.

Number of Bull's-Eyes Made	Points Displayed
0	12,000
3	36,000
5	52,000
9	84,000

2.

x	y
6	12
−4	7
−12	−3
−22	−8

Determine whether the slope of the line represented by each equation is positive, negative, zero, or undefined.

3. $y = -x + 5$

4. $x = 0$

In the figure, parallel lines r and s are cut by transversal w.

5. List all pairs of corresponding angles.

6. List all pairs of alternate interior angles.