

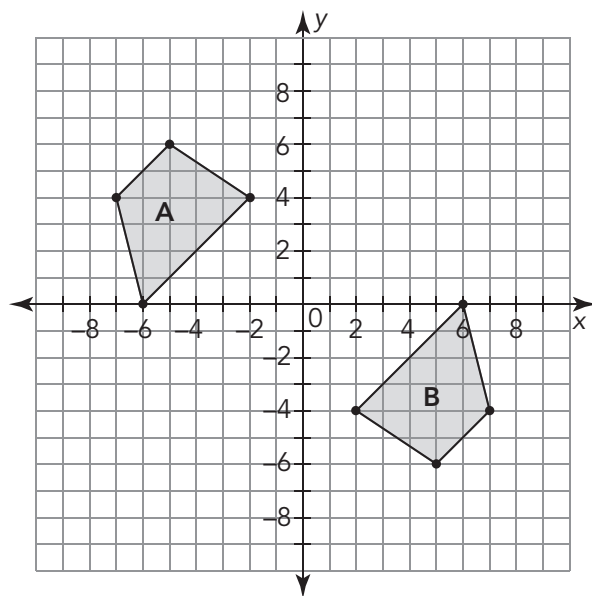
# From Here to There

## 3

### Mapping Similar Figures Using Transformations

#### WARM UP

1. Describe at least two different single transformations or sequences of transformations that map Figure A to Figure B.
2. Describe the geometric relationships between the figures.



#### LEARNING GOALS

- Describe a single dilation that maps a two-dimensional figure onto a similar figure.
- Determine a sequence of transformations that maps a two-dimensional figure onto a similar figure.
- Determine the relationship between images of the same pre-image.

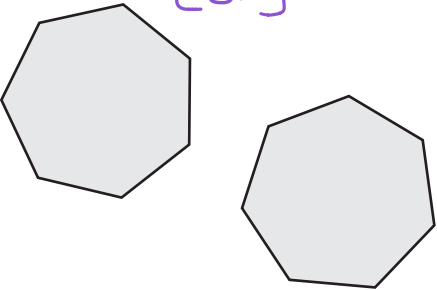
You have used sequences of translations, reflections, and rotations to verify that two images are congruent. How can you use transformations to determine if two images are similar and/or congruent?

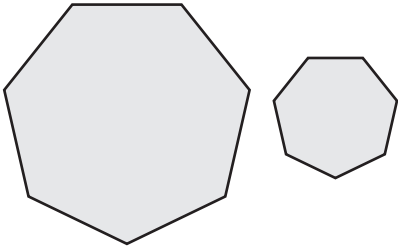
## Same Figure or Same Shape?

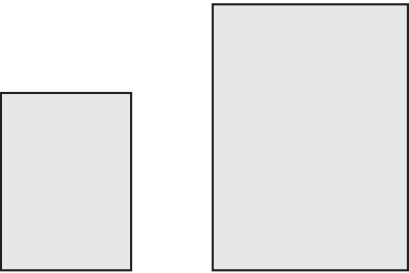
When two figures are similar, the same scale factor can be applied to all side lengths to map one figure to the other.

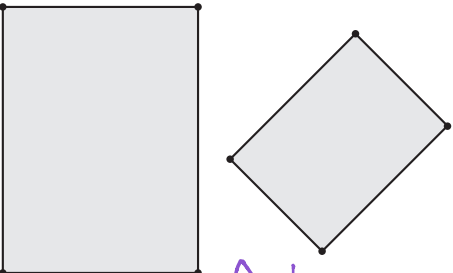
We often say that dilations preserve shape and that rigid motions preserve both size and shape. As a result, it is common to state that similar figures have the same shape, and congruent figures have the same size and shape. However, what does it mean for two figures to have the same shape in this context? Are all rectangles similar? Are all triangles similar?

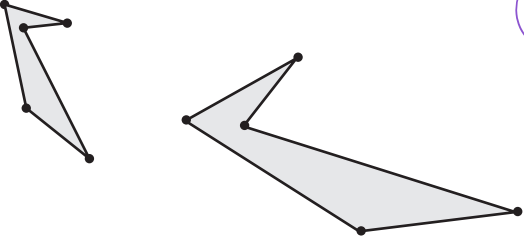
Use the definition of similar figures to determine which figures are similar.

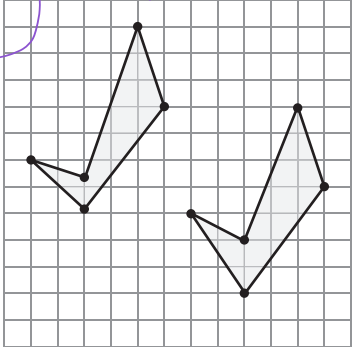
1.  *congruent* → *similar*

2. 

3. 

4. 

5. 

6.  *No*

Do you think all rectangles are similar to each other? What about squares?



ACTIVITY  
**3.1**

# Proving Similarity Through Dilations



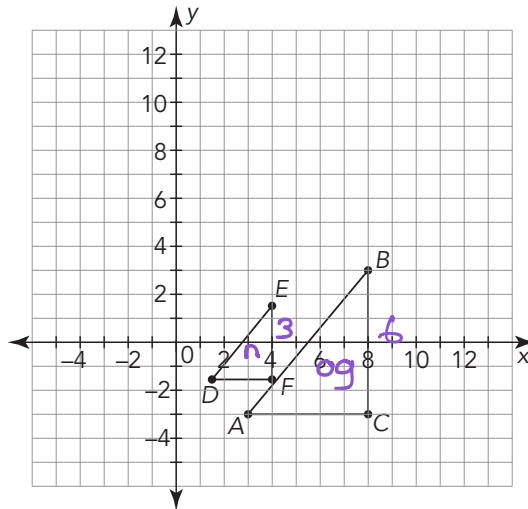
NOTES

In this activity, you will use what you know about dilations to determine if figures are similar.

1. Determine if the figures are similar. If they are similar, state the scale factor and the center of dilation that maps Figure 1 onto Figure 2.

a. Figure 1:  $\triangle ABC$

Figure 2:  $\triangle DEF$



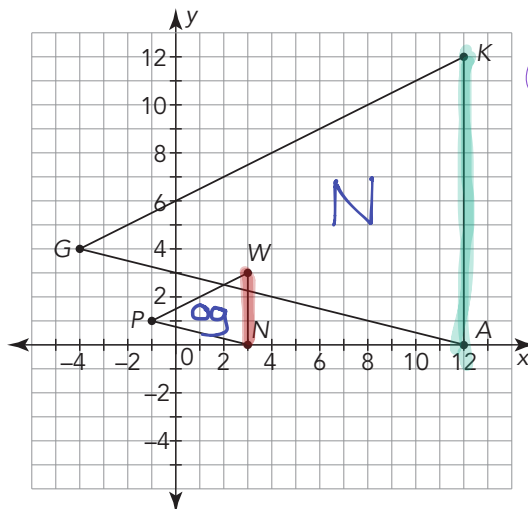
center (0,0)

scale factor:

$$\frac{\text{new}}{\text{original}} = \frac{EF}{BC} = \frac{3}{2}$$

b. Figure 1:  $\triangle PWN$

Figure 2:  $\triangle GKA$



center (0,0)

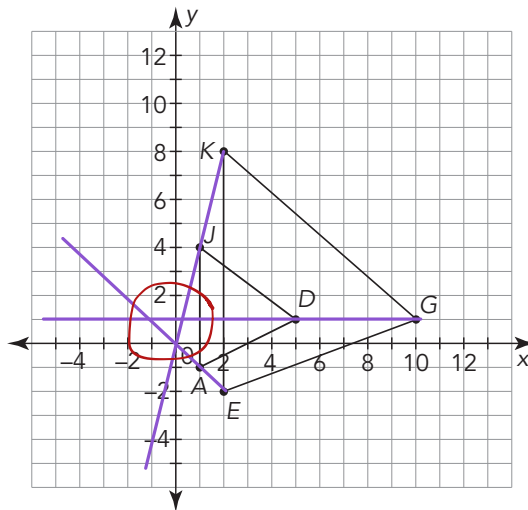
Scale factor:

$$\frac{\text{new}}{\text{original}} = \frac{KA}{WN} = \frac{12}{3}$$

$$= 4$$

c. Figure 1:  $\triangle JDA$

Figure 2:  $\triangle KGE$

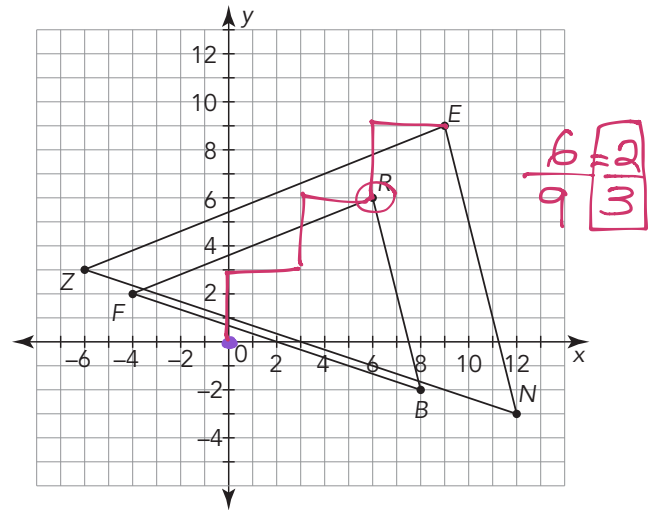


Not similar

d. Figure 1:  $\triangle ZEN$

Figure 2:  $\triangle FRB$

Center  $(0,0)$   
Scale factor:  $\frac{2}{3}$

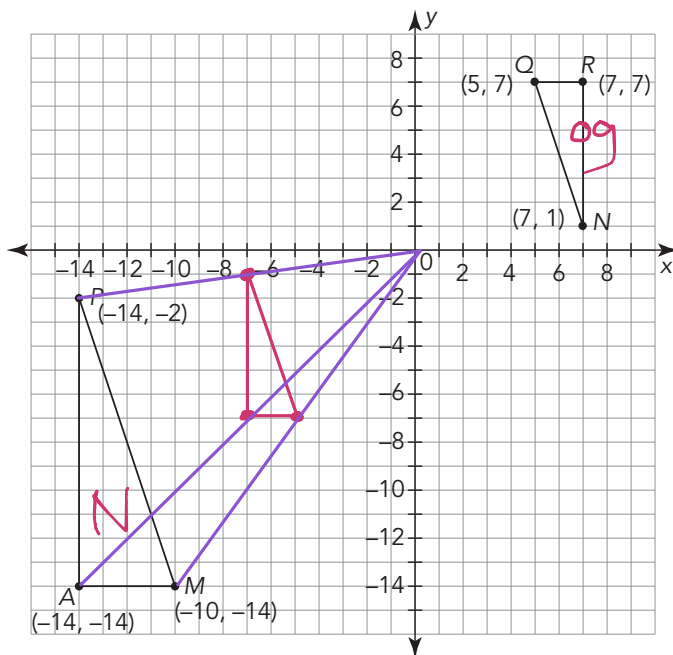


## ACTIVITY 3.2 Proving Similarity Through Transformations



Sometimes similar figures cannot be mapped from one to another using only a dilation. You may need a combination of translations, reflections, rotations, and dilations to map a figure onto a similar figure.

1. Triangle  $MAP$  is the image of Triangle  $QRN$  after undergoing at least one transformation.



a. Determine a possible sequence of transformations to map  $\triangle QRN$  onto  $\triangle MAP$ .

- 180° rotation about  $(0,0)$   
- Dilate by 2 about  $(0,0)$

b. Are the triangles congruent? Are they similar? Explain your reasoning.

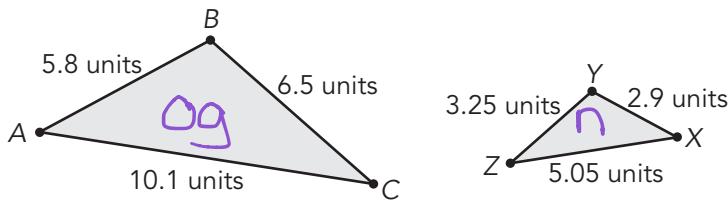
No, they are not congruent. They are similar (same shape)

diff size)

- c. Reverse the order of the sequence of transformations you described in part (b). What do you notice?

The order of transformations does not matter.

2. Triangle XYZ is the image of Triangle ABC after undergoing at least one transformation.



- a. List the corresponding sides and angles for  $\triangle ABC$  and  $\triangle XYZ$ .

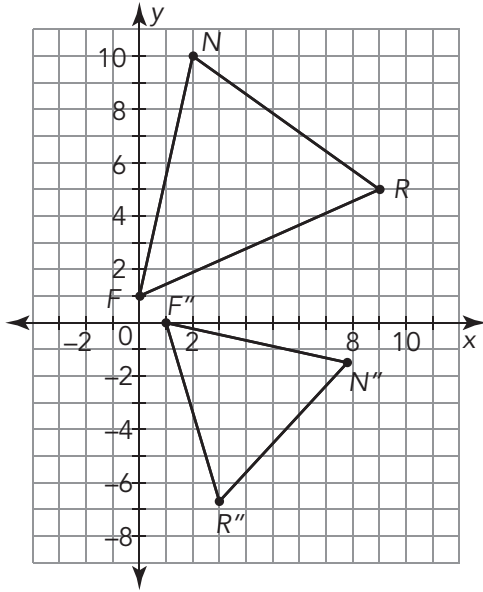
$$\frac{\text{new}}{\text{original}} = \frac{XY}{AB} = \frac{2.9}{5.8} \quad \frac{ZX}{CA} = \frac{5.05}{10.1}$$
$$= \frac{YZ}{BC} = \frac{3.25}{6.5} = \boxed{\frac{1}{2}}$$

- b. Determine a possible sequence of transformations to map  $\triangle ABC$  onto  $\triangle XYZ$ .

reflection  
dilation

- c. Reverse the order of the sequence of transformations you described in part (b). What do you notice?

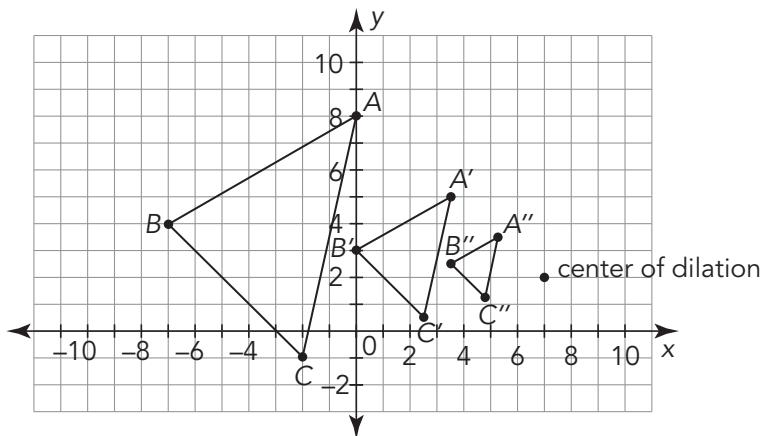
3. Triangle  $F''N''R''$  is the image of Triangle  $FNR$  after two transformations.



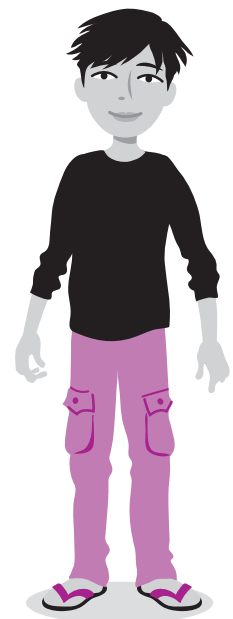
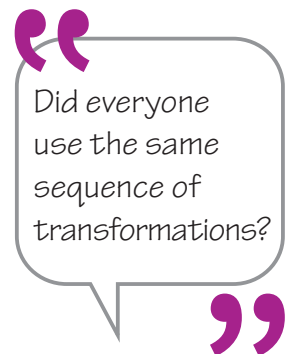
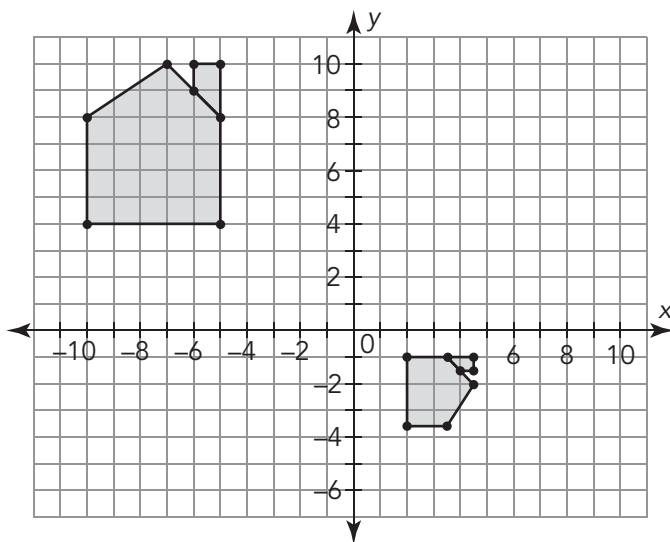
a. Determine a possible sequence of a rotation and dilation to map  $\triangle FNR$  onto  $\triangle F''N''R''$ .

b. Reverse the order of the sequence of transformations you described in part (a). Explain any adjustments you need to make in the sequence of transformations to create a correct mapping.

4. Triangle  $ABC$  was dilated to create Triangle  $A'B'C'$ . Then Triangle  $A'B'C'$  was dilated to create Triangle  $A''B''C''$ . Describe a single transformation that maps  $\triangle ABC$  onto  $\triangle A''B''C''$ .



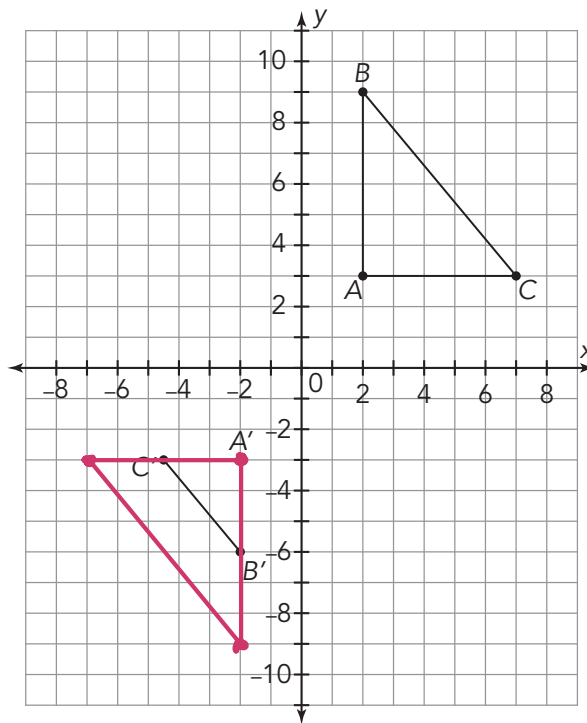
5. Verify that the two houses are similar by describing a sequence of transformations that maps one figure onto the other.



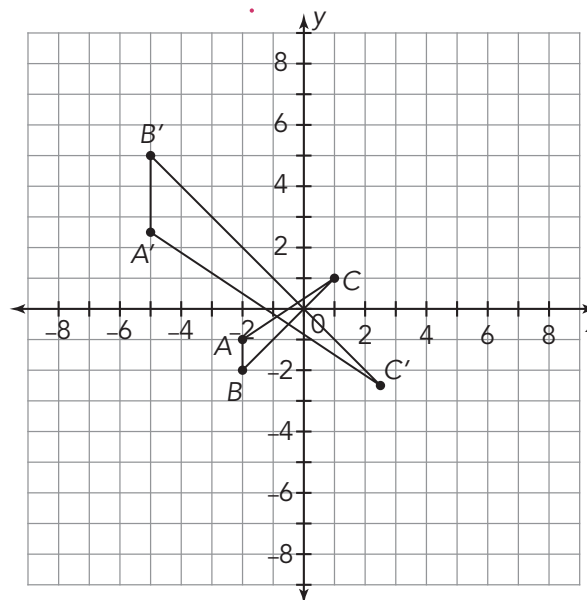
6. Use dilations and other transformations to determine if the triangles represented by the coordinates are similar. Show your work and explain your reasoning.

- a.  $A(2, 3)$   $B(2, 9)$   $C(7, 3)$   
 $A'(-2, -3)$   $B'(-2, -6)$   $C'(-4.5, -3)$

Rotate  $180^\circ$   
 about  $(0, 0)$   
 - Dilate w/  
 scale factor  
 of  $\frac{1}{2}$   
 about point A

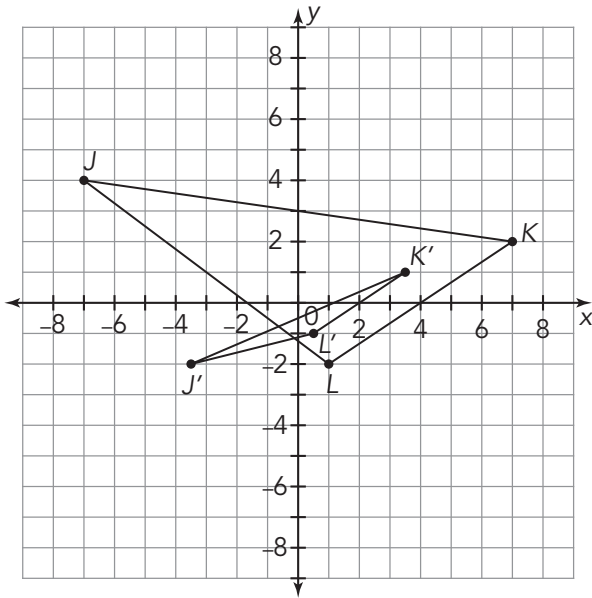


- b.  $A(-2, -1)$   $B(-2, -2)$   $C(1, 1)$   
 $A'(-5, 2.5)$   $B'(-5, 5)$   $C'(2.5, -2.5)$

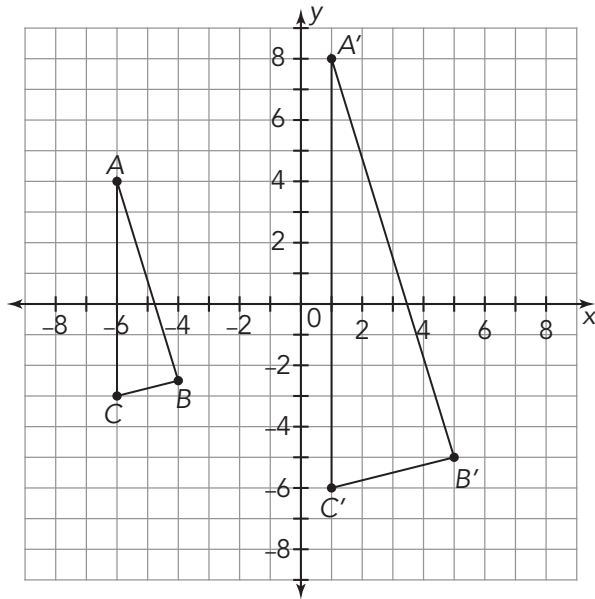




- c.  $J(-7, 4)$   $K(7, 2)$   $L(1, -2)$   
 $J'(-3.5, -2)$   $K'(3.5, 1)$   $L'(0.5, -1)$



- d.  $A(-6, 4)$   $B(-4, -2.5)$   $C(-6, -3)$   
 $A'(1, 8)$   $B'(5, -5)$   $C'(1, -6)$



ACTIVITY  
3.3

# Comparing Images



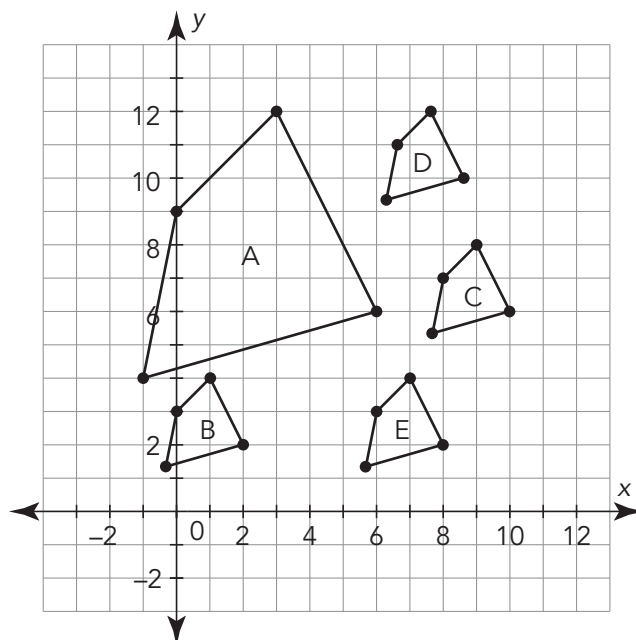
“  
Make a  
conjecture!  
”



You know that similar figures can be mapped from one to another using a sequence of transformations. How are the images of the same pre-image related to each other?

Let's investigate!

1. **Quadrilateral A is the pre-image used to create Quadrilaterals B, C, D, and E using dilations.**



“  
How can you  
verify your  
results?  
”

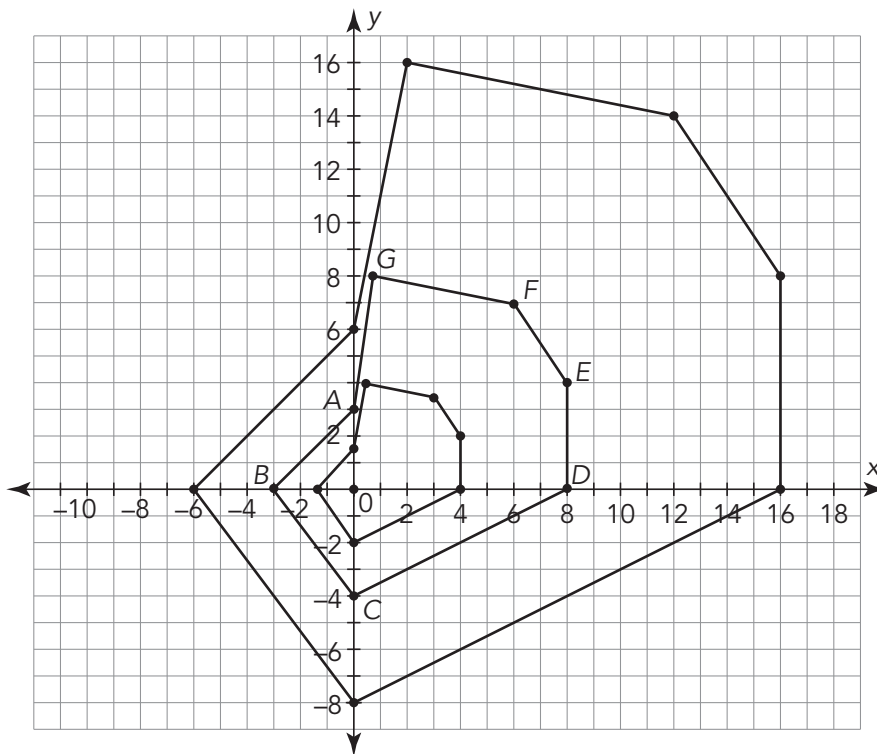


- a. **Determine the scale factor used to map Quadrilateral A onto each of the other quadrilaterals. Explain your reasoning.**
- b. **Was the same center of dilation used to create each of the other quadrilaterals? Explain your reasoning.**

- c. Are Quadrilaterals  $B$ ,  $C$ ,  $D$ , and  $E$  similar? Are they congruent? Explain your reasoning.

2. The labeled figure is the pre-image used to create the other two figures using dilations.

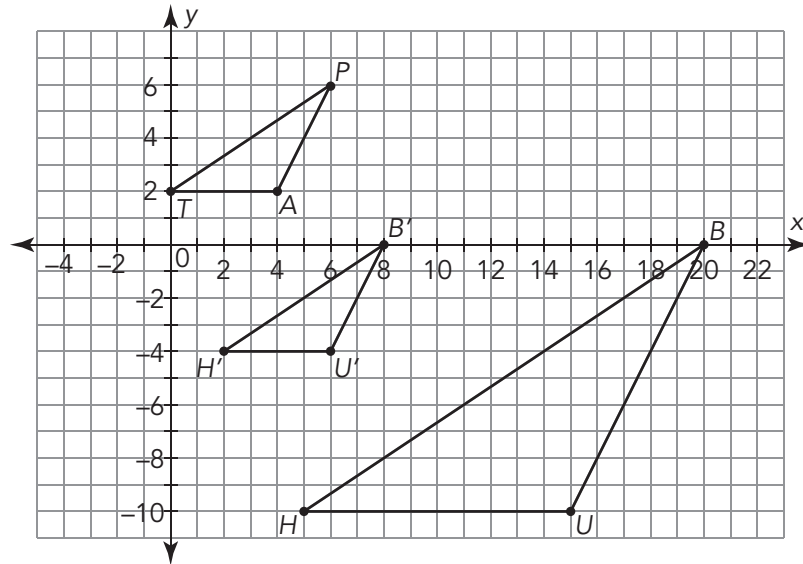
- a. Determine the scale factor to map the pre-image to each of the other figures. Explain your reasoning.



- b. Was the same center of dilation used to create each of the other figures? Explain your reasoning.

- c. Are the images similar? Are they congruent? Explain your reasoning.

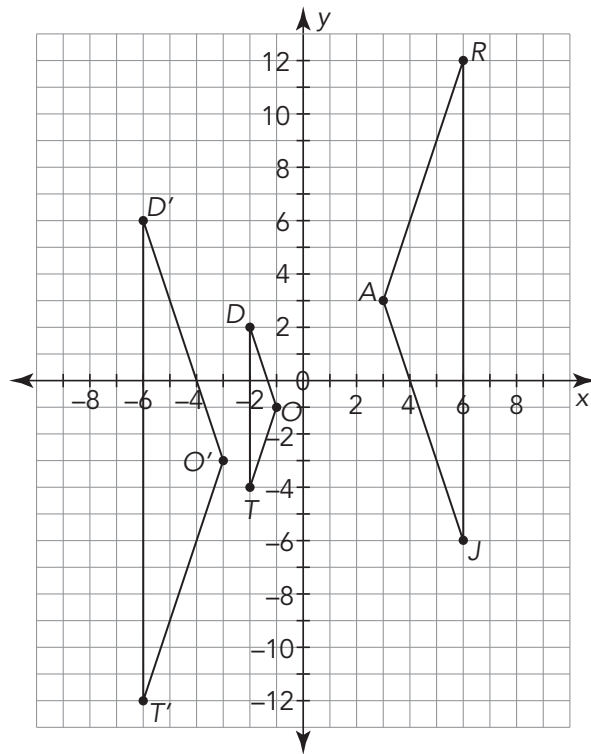
3. Triangle  $HUB$  was dilated from the origin by a scale factor of  $\frac{2}{5}$  to create  $\triangle H'U'B'$ , and  $\triangle H'U'B' \cong \triangle TAP$ .



- a. What is the relationship between  $\triangle HUB$  and  $\triangle TAP$ ? Justify your answer.

- b. Determine a possible sequence of transformations that maps  $\triangle HUB$  onto  $\triangle TAP$ .

4. Triangle  $DOT$  was dilated from the origin by a scale factor of 3 to create  $\triangle D'O'T'$ , and  $\triangle D'O'T' \cong \triangle JAR$ . Determine a possible sequence of transformations that maps  $\triangle JAR$  onto  $\triangle DOT$ .



## TALK the TALK

### Summing Up Similar Figures

Determine if each statement is *always*, *sometimes*, or *never* true. Provide a justification for each answer.

1. Triangle  $ABC$  is dilated four times with different scale factors and different centers of dilation. The four images are congruent.
2. Triangle  $HIP$  is dilated by a scale factor of 8, followed by a scale factor of 0.125. The final image is congruent to  $\triangle HIP$ .
3. The same order for a sequence of transformations can be used to map between two similar figures, regardless of which figure is used as the pre-image.
4. Dilations are used to create congruent figures.
5. Transformations are used to create similar figures.

# Assignment

## Write

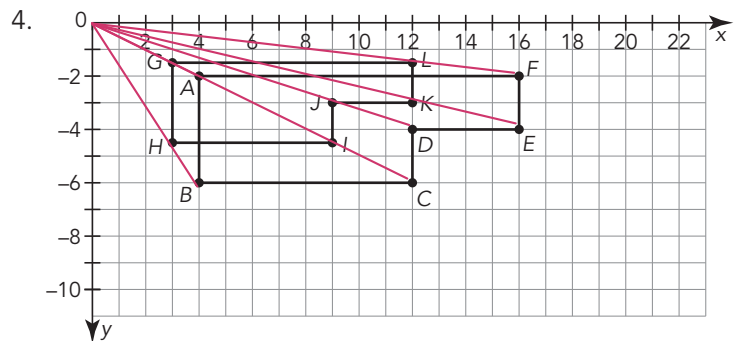
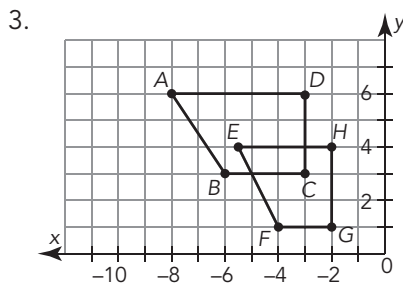
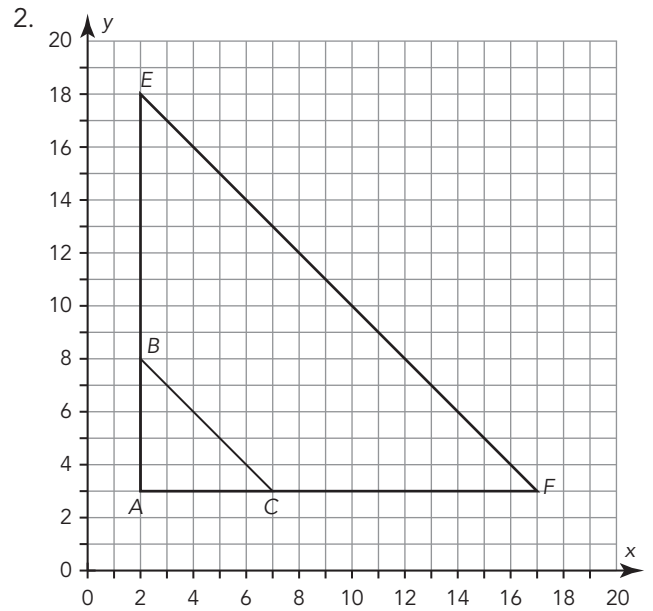
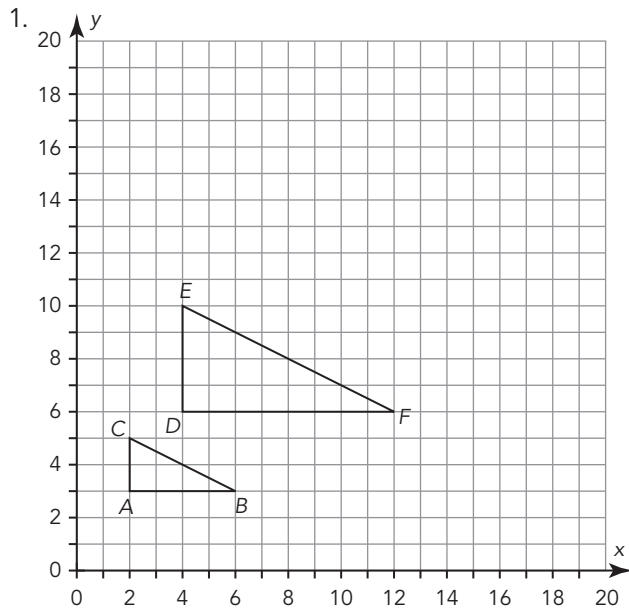
Explain how to use transformations to determine if figures are congruent or similar.

## Remember

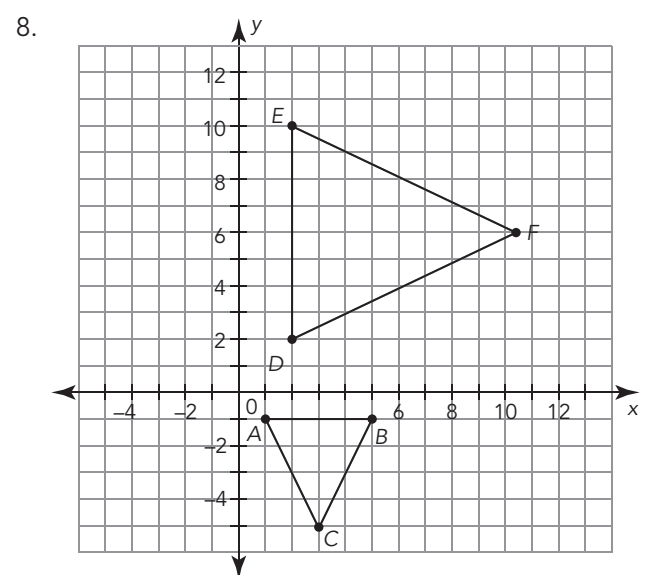
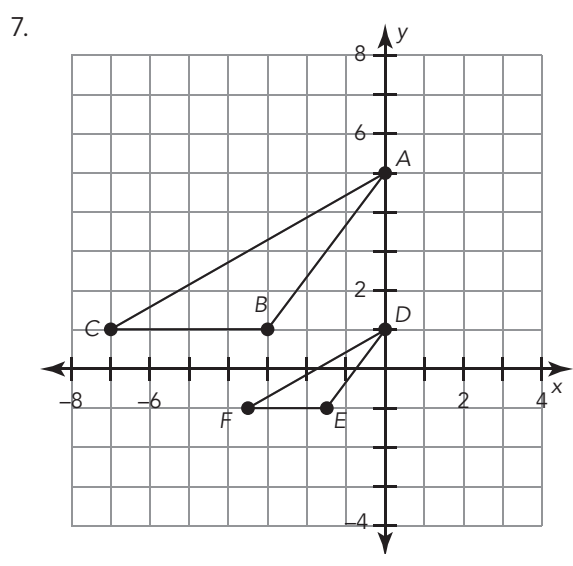
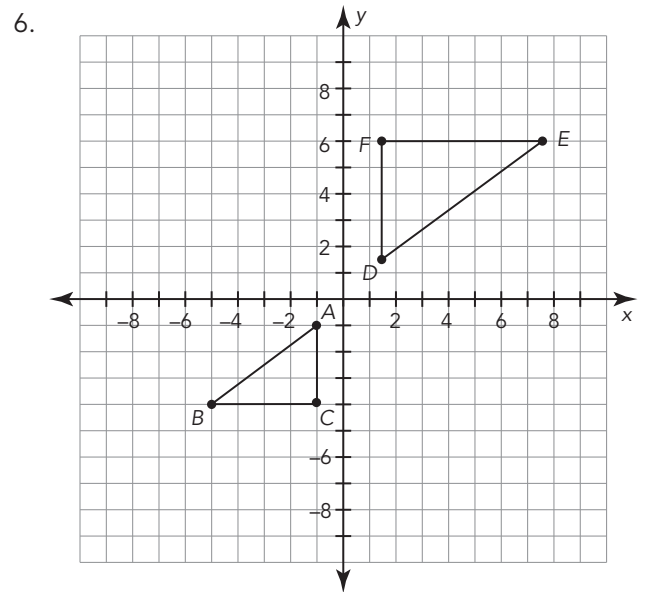
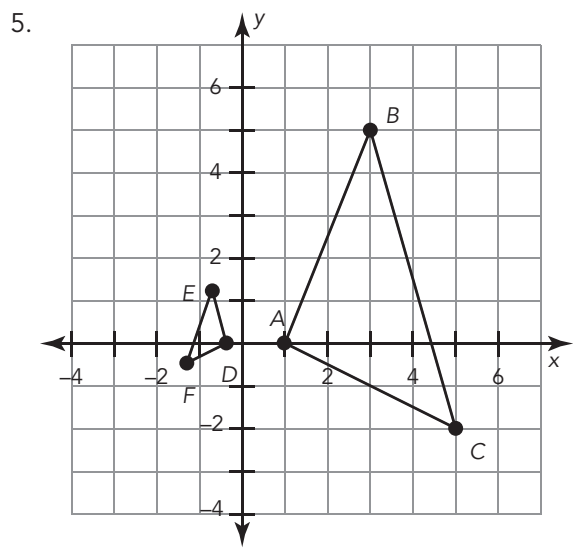
Images created from the same pre-image are always similar figures.

## Practice

Verify that the two figures are similar by describing a dilation that maps one figure onto the other. Be sure to include the scale factor.



Verify that the figures are similar by describing a sequence of transformations that maps  $\triangle ABC$  onto  $\triangle DEF$ . Be as specific as possible.

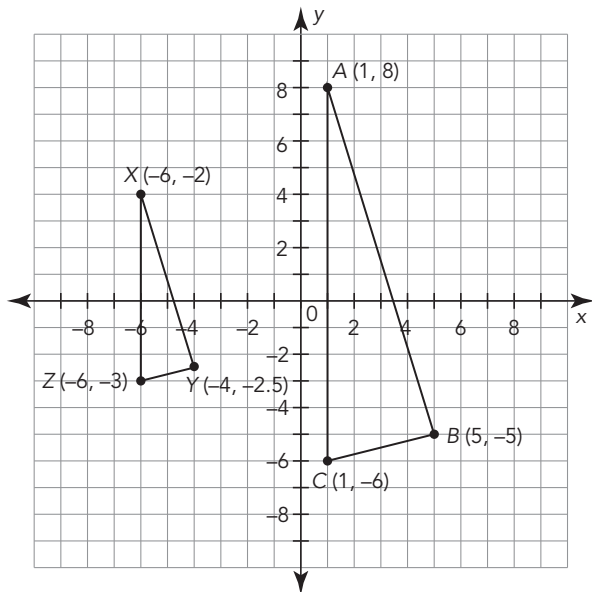




## Stretch

Triangle  $XYZ$  is the image after a dilation of Triangle  $ABC$ .

1. Determine the scale factor.
2. Determine the center of dilation.
3. Explain how you could verify that the ratio of corresponding sides is constant.



## Review

1. Give the coordinates of  $\triangle A'B'C'$  after a transformation of  $\triangle ABC$  with the coordinates  $A(6, -3)$ ,  $B(9, 5)$ , and  $C(5, 6)$ . Use the origin as the center of dilation or rotation, as needed.
  - a. Dilate  $\triangle ABC$  by a scale factor of  $\frac{1}{3}$ .
  - b. Dilate  $\triangle ABC$  by a scale factor of 4.
  - c. Rotate  $\triangle ABC$  180 degrees.
  - d. Reflect  $\triangle ABC$  across the x-axis.
2. Identify the constant of proportionality.
  - a. Eight candy bars cost \$6.00. Calculate the cost per candy bar.
  - b. In the equation  $y = 4x + 7$ ,  $x$  is the number of items and  $y$  is the total cost. What is the unit rate? Include units in your response.

