One or More Xs to One Y

3

Defining Functional Relationships

WARM UP

Evaluate each expression given the set of values {1, 6, 12, 25}.

1. $5x$

2. $\frac{1}{2}x + 1$

3. $x - 8$

LEARNING GOALS

- Describe a functional relationship in terms of a rule which assigns to each input exactly one output.
- Determine whether a relation (represented as a mapping, set of ordered pairs, table, sequence, graph, equation, or context) is a function.

KEY TERMS

- mapping
- set
- relation
- input
- output
- function
- domain
- range
- scatter plot
- vertical line test

Throughout middle school, you have investigated different types of relationships between variable quantities: additive, multiplicative, proportional, and non-proportional. What are functional relationships?

Getting Started

What's My Rule?

Rules can be used to generate sequences of numbers. They can also be used to generate (*x*, *y*) ordered pairs.

$x \rightarrow \boxed{\text{rule}} \rightarrow y$

1. Write an equation to describe the relationship between each independent variable *x* and the dependent variable *y*. Explain your reasoning.

$\boxed{y = 4x + 12}$

$= 4(3) + 12$

$= 12 + 12$

$= 24$ ✓

a.

x	y
−6	−12
−3	0
0	12
3	24

$m = \frac{12-0}{0-(-3)} = \frac{12}{3} = 4$

b.

x	y
1	−2
5	−10
−1	2
−10	20

c.

x	y
−10	9
−2	1
0	−1
5	4

d.

x	y
0	2
4	4
5	4.5
20	12

$m = \frac{4-2}{4-0} = \frac{2}{4} = \frac{1}{2}$

$y = \frac{1}{2}x + 2$

2. Create your own table and have a partner determine the equation you used to build it.

ACTIVITY
3.1

Functions as Mappings from One Set to Another

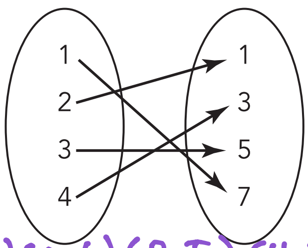
As you learned previously, ordered pairs consist of an *x*-coordinate and a *y*-coordinate. You also learned that a series of ordered pairs on a coordinate plane can represent a pattern. You can also use a *mapping* to show ordered pairs. A **mapping** represents two sets of objects or items. Arrows connect the items to represent a relationship between them.

When you write the ordered pairs for a mapping, you are writing a set of ordered pairs. A **set** is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.

Use braces, { }, to denote a set.

1. Write the set of ordered pairs that represent a relationship in each mapping.

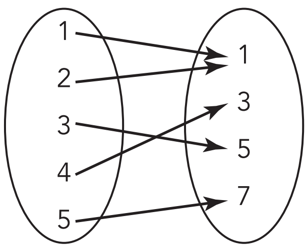
a.



{(1,7)(2,1)(3,5)(4,3)}

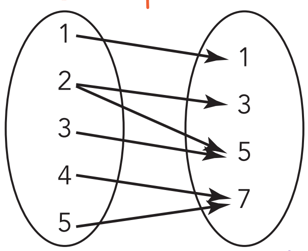
set

b.



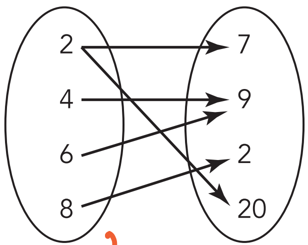
{(1,1)(2,1)(3,5)(4,3)(5,7)}

c.



{(1,1)(2,3)(2,5)(3,5)(4,7)(5,7)}

d.

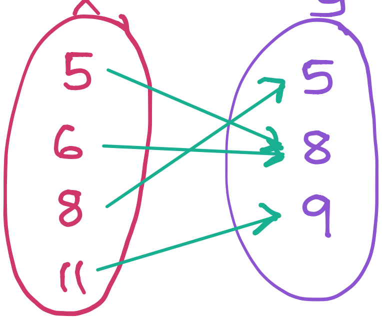


{(2,7)(2,20)(4,9)(6,9)(8,2)}

2. Create a mapping from the set of ordered pairs.

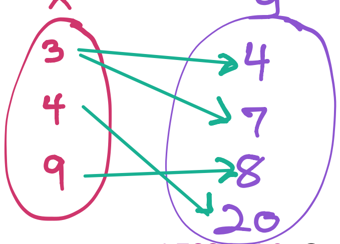
a. {(5, 8), (11, 9), (6, 8), (8, 5)}

x y



b. {(3, 4), (9, 8), (3, 7), (4, 20)}

x y



(x , y)
↓ ↓
input output
↓ ↓
domain (x-values) range (y-values)

3. Write the set of ordered pairs to represent each table.

a.

Input	Output
−10	−20
−5	−10
0	0
5	10
10	20

b.

x	y
20	−10
10	−5
0	0
10	5
20	10

{(-10,-20)(-5,-10)(0,0)(5,10)(10,20)}

Set → a group of ordered pairs

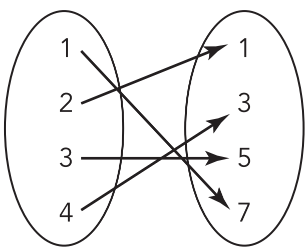
The mappings and ordered pairs shown in Questions 1 through 3 form *relations*. A **relation** is any set of ordered pairs or the mapping between a set of *inputs* and a set of *outputs*. The first coordinate of an ordered pair in a relation is the **input**, and the second coordinate is the **output**. A **function** maps each input to one and only one output. In other words, a function has no input with more than one output. The **domain** of a function is the set of all inputs of the function. The **range** of a function is the set of all outputs of the function.

* One x to One y

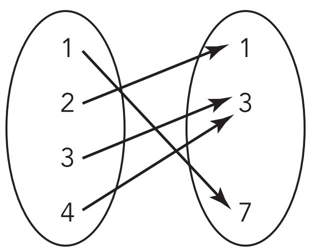
Notice the use of set notation when writing the domain and range.

WORKED EXAMPLE

In each mapping shown, the domain is {1, 2, 3, 4}.



The range is {1, 3, 5, 7}.



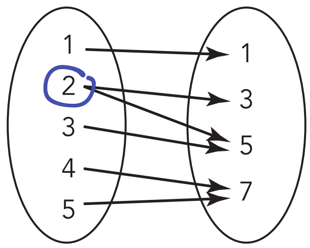
The range is {1, 3, 7}.

Each mapping represents a function because no input, or domain value, is mapped to more than one output, or range value.

NOTES

WORKED EXAMPLE

In the mapping shown, the domain is {1, 2, 3, 4, 5} and the range is {1, 3, 5, 7}.



This mapping does not represent a function.

4. State why the relation in the worked example shown is not a function.

Not a function because

not one-to-one

$2 \to 3$ (2, 3)

$2 \to 5$ (2, 5)

5. State the domain and range for each relation in Questions 2 and 3. Then, determine which relations represent functions. If the relation is not a function, explain why not.

#2 a Domain: {5, 6, 8, 11}

Range: {5, 8, 9}

Function.

b Domain: {3, 4, 9}

Range: {4, 7, 8, 20}

Not a function

#3 a Domain: {-10, -5, 0, 5, 10}

Range: {-20, -10, 0, 10, 20}

Function

b Domain: {0, 10, 20}

Range: {-10, -5, 0, 5, 10}

Not a function

Yes

$5 \to 8$

$6 \to 8$

$7 \to 8$

one-to-one

No

$5 \to 8$

$5 \to 9$

$5 \to 10$

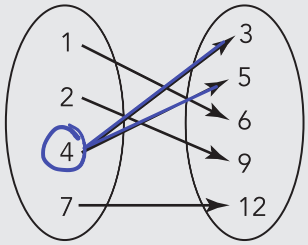
not one-to-one

Think about the mappings as ordered pairs.

6. **Review and analyze Emil's work. Explain why Emil's mapping is not an example of a function.**

Emil

My mapping represents a function.



Not a function because 4 maps to 3 & 5

$4 \to 3$
$4 \to 5$

7. **Determine if each sequence represents a function. Explain why or why not. If it is a function, identify its domain and range. Create a mapping to verify your answer.**

   a. **2, 4, 6, 8, 10, …**

Remember that a sequence has a term number and a term value.

   b. **1, 0, 1, 0, 1, …**

   c. **0, 5, 10, 15, 20, …**

ACTIVITY
3.2

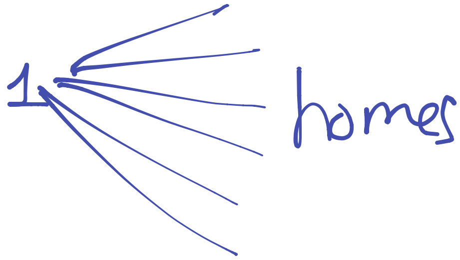
Functions as Mapping Inputs to Outputs

You have determined if sets of ordered pairs represent functions. In this activity you will examine different situations and determine whether they represent functional relationships.

Read each context and decide whether it fits the definition of a function. Explain your reasoning.

Yes
1. *Input:* **Sue writes a thank-you note to her best friend.**
   Output: **Her best friend receives the thank-you note in the mail.**

No
2. *Input:* **A football game is being telecast.**
   Output: **It appears on televisions in millions of homes.**

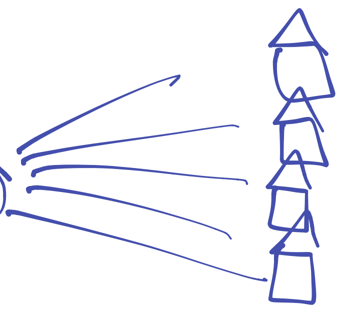


Yes
3. *Input:* **There are four puppies in a litter.**
   Output: **One puppy was adopted by the Smiths, another by the Jacksons, and the remaining two by the Fullers.**

1 → S
2 → J
3 → F
4 → F

Yes
4. *Input:* **The basketball team has numbered uniforms.**
   Output: **Each player wears a uniform with her assigned number.**

No
5. *Input:* **Beverly Hills, California, has the zip code 90210.**
   Output: **There are 34,675 people living in Beverly Hills.**



No
6. *Input:* **A sneak preview of a new movie is being shown in a local theater.**
   Output: **65 people are in the audience.**

7. *Input:* Tara works at a fast food restaurant on weekdays and a card store on weekends.
Output: Tara's job on any one day.

M
Tu
W
Th
F
Sat
Sun
R
C

Yes

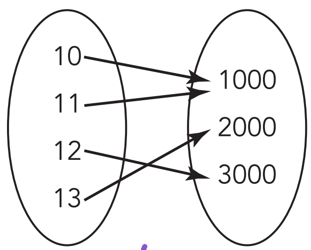
8. *Input:* Janelle sends a text message to everyone in her contact list on her cell phone.
Output: There are 41 friends and family on Janelle's contact list.

No

ACTIVITY 3.3 Determining Whether a Relation Is a Function

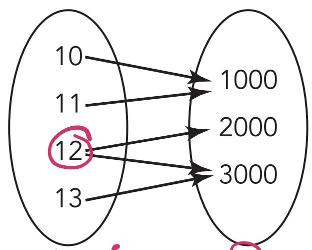
Analyze the relations in each pair. Determine which relations are functions and which are not functions. Explain how you know.

1. **Mapping A**



Function
Domain: {10, 11, 12, 13}
Range: {1000, 2000, 3000}

Mapping B



Not a function
Domain: {10, 11, 12, 13}
Range: {1000, 2000, 3000}

2. Table A

Function

Input	Output
−2	4
−1	1
0	0
1	1
2	4

Domain: {-2, -1, 0, 1, 2} Range: {0, 1, 4}

Table B

x	y
2	−4
1	−1
0	0
1	1
2	4

Not a function

Domain: {0, 1, 2}

Range {-4, -1, 0, 1, 4}

3. Sequence A

7, 10, 13, 16, 19, ...

Sequence B

10, 30, 10, 30, 10, ...

4. Set A

{(2, 3), (2, 4), (2, 5), (2, 6), (2, 7)}

Not a function.

Domain: {2}

Range: {3, 4, 5, 6, 7}

Set B

{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1)}

Function

Domain: {2, 3, 4, 5, 6} Range: {1}

5. Scenario A

Input:

The morning announcements are read over the school intercom system during homeroom period.

Output:

All students report to homeroom at the start of the school day to listen to the announcements.

Not a function

1 to 35

Scenario B

Input:

Each student goes through the cafeteria line.

Output:

Each student selects a lunch option from the menu.

Function.

ACTIVITY 3.4

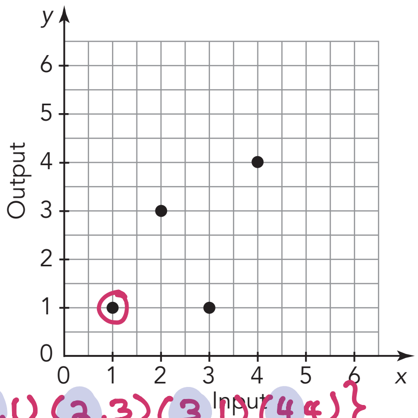
Functions as Graphs

A relation can be represented as a graph.

A **scatter plot** is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.

1. Determine if each scatter plot represents a function. Explain your reasoning.

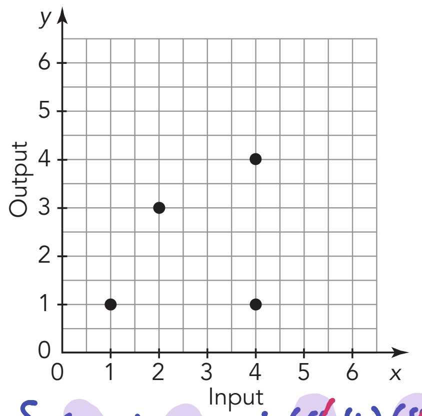
a.



{(1,1) (2,3) (3,1) (4,4)}

Function

b.



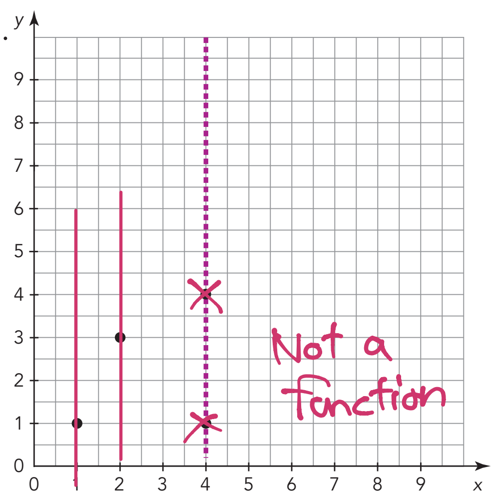
{(1,1) (2,3) (4,4) (4,1)}

Not a function

The **vertical line test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the vertical line test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

WORKED EXAMPLE

Consider the scatter plot shown.



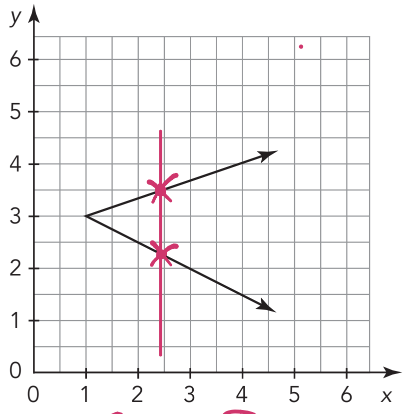
In this scatter plot, the relation is not a function. The input value 4 can be mapped to two different outputs, 1 and 4. Those two outputs are shown as intersections to the vertical line drawn at $x = 4$.

2. Use the definition of function to explain why the vertical line test works.

Vertical line test – A relation is a function if the vertical line crosses each point one at a time.

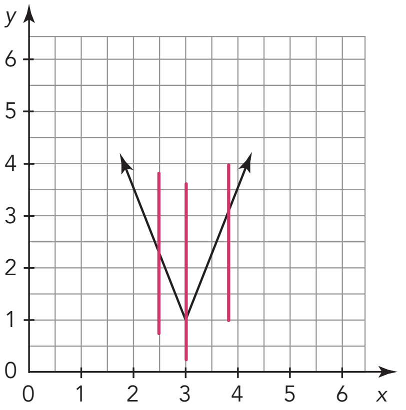
3. Use the vertical line test to determine if each graph represents a function. Explain your reasoning.

a.



Not a function.

b.



Function.

4. Use the 12 cards that you sorted in the previous lesson. Sort the graphs into two groups: functions and non-functions. Use the letter of each graph to record your findings.

Functions	Non-functions
H, B, L, E K, C, G, A	I, F, J, D

NOTES

NOTES

ACTIVITY 3.5 Functions as Equations

So far, you have determined whether a mapping, context, or a graph represents a function. You can also determine whether an equation is a function.

WORKED EXAMPLE

The given equation can be used to convert yards to feet. Let x represent the number of yards, and let y represent the number of feet.

$$y = 3x$$

To test whether this equation is a function, first, substitute values for x into the equation, and then determine if any x-value can be mapped to more than one y-value. If each x-value has exactly one y-value, then it is a function. Otherwise, it is not a function.

x	$y = 3x$
1	3
3	9
4	12
8	24

In this case, every x-value can be mapped to only one y-value. Each x-value is multiplied by 3. Some examples of ordered pairs are (2, 6), (10, 30), and (5, 15). Therefore, this equation is a function.

It is not possible to test every possible input value in order to determine whether or not the equation represents a function. You can graph any equation to see the pattern and use the vertical line test to determine if it represents a function.

1. **Determine whether each equation is a function. List three ordered pairs that are solutions to each. Explain your reasoning.**

If you do not recognize the graph of the equation, use a graphing calculator to see the pattern.

a. $y = 5x + 3$

y=mx+b

linear equation/function

Function

b. $y = x^2$

Quadratic

(parabola)

Function

c. $y = |x|$

Absolute value equation

Function

d. $x^2 + y^2 = 1$

Circle

Not a Function

e. $y = 4$

horizontal line

Function

f. $x = 2$

vertical line

Not a Function

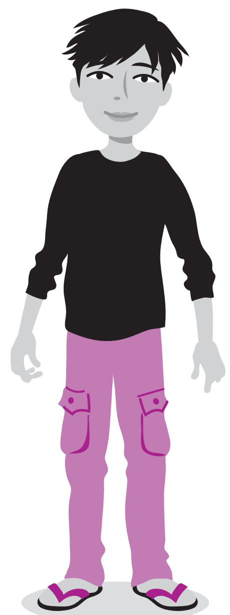
2. **Explain what is wrong with Taylor's reasoning.**

Taylor

The equation $y^2 = x$ represents a function.

x	y
4	2
9	3
25	5

If two different inputs go to the same output, it can still be a function.



NOTES

TALK the TALK

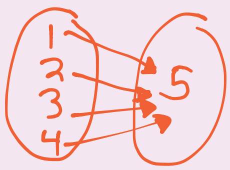
Function Organizer

1. Complete the graphic organizer for the concept of function. Write a definition for *function* in your own words. Then, create a problem situation that can be represented using a function. Finally, create a table of ordered pairs and sketch a graph to represent the function.

Function

Definition

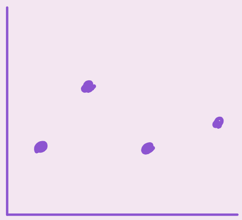
One x to one y (1-to-1)



Problem Situation

- one capital to one state
- one president to one country
- one flavor to one cone.
- one license plate # to one car.

Graph



discrete

continuous

Table/Ordered Pairs

x	y
1	5
2	5
3	5
4	5

{(1,5) (2,5) (3,5) (4,5)}

Assignment

Write

Write the term from the box that best completes each sentence.

scatter plot	output	relation	input	vertical line test
mapping	set	domain	range	function

1. A(n) ______________ is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.
2. The first coordinate of an ordered pair in a relation is the ______________.
3. The second coordinate of an ordered pair is the ______________.
4. A(n) ______________ maps each input to one and only one output.
5. A(n) ______________ is a graph of a collection of ordered pairs.
6. The ______________ is a visual method of determining whether a relation represented as a graph is a function by visualizing whether any vertical lines would intersect the graph of the relation at more than one point.
7. A(n) ______________ shows objects in two sets connected together to represent a relationship between the two sets.
8. A(n) ______________ is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.
9. The ______________ of a function is the set of all inputs of the function.
10. The ______________ of a function is the set of all outputs of the function.

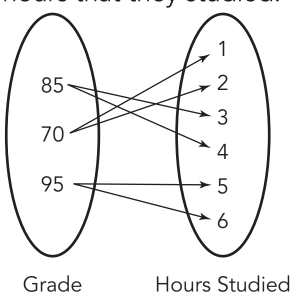
Remember

A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.

A relation is a function when each input value maps to one and only one output value.

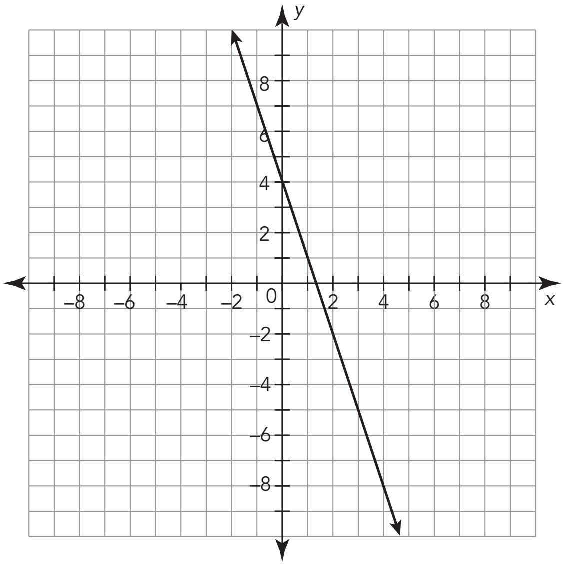
Practice

1. A history teacher asks six of her students the number of hours that they studied for a recent test. The diagram shown maps the grades that they received on the test to the number of hours that they studied.
   a. Is the relation a function? If the relation is not a function, explain why not.
   b. Write the set of ordered pairs to represent the mapping.
   c. What does the first value in each ordered pair in part (b) represent? What does the second value in each ordered pair represent?
   d. Create a scatter plot. Does the graph agree with your conclusion from part (a)? Explain your reasoning.

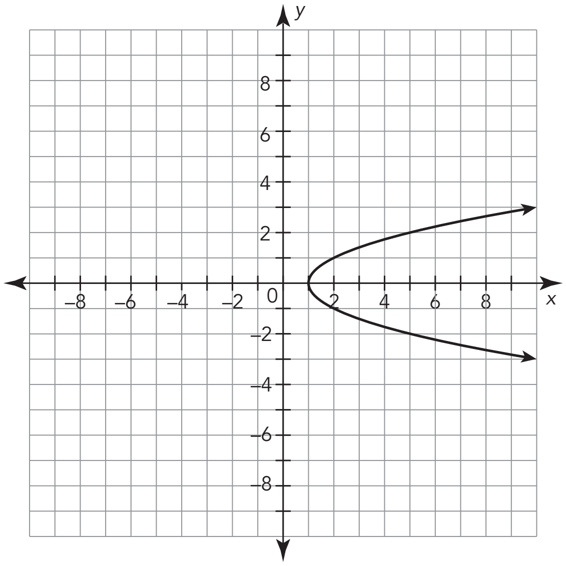


2. The science teacher created the set of ordered pairs {(100, 6), (90, 5), (80, 3), (70, 1), (90, 4), (80, 2)} to represent six students' grades on the midterm to the number of hours that they had studied. Create a mapping from this set of ordered pairs.
   a. Is the relation a function? If the relation is not a function, explain why not.
   b. List all the inputs of the relation.
   c. List all the outputs of the relation.
   d. Instead of mapping grades to hours studied, the teacher decides to create a new diagram. This diagram maps hours studied to grades. Show the mapping that would result.
   e. Write the set of ordered pairs to represent the mapping in part (d).
   f. Is the relation in part (d) a function? If the relation is not a function, explain why not.
   g. Create a scatter plot. Does the graph agree with your conclusion from part (f)? Explain your reasoning.
3. At the end of the year, a principal decides to create the given mapping.
   Input: the 82 total students in the history class
   Ouput: the final grades they received for the class
   Does this mapping fit the definition of a function? Explain your reasoning.
4. Use the vertical line test to determine if each graph represents a function. Explain your reasoning.

a.



b.



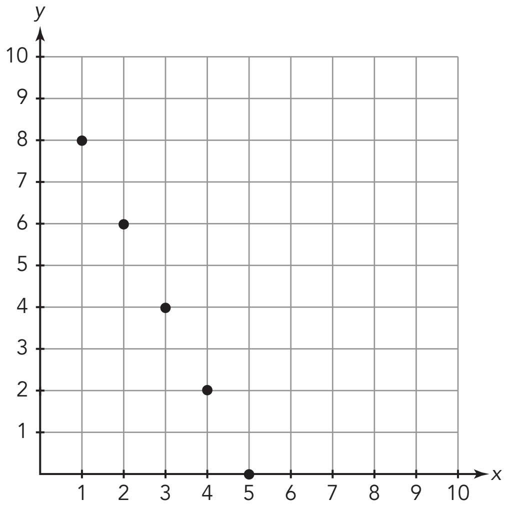
Stretch

Describe how you can tell from an equation whether a function is increasing, decreasing, or constant.

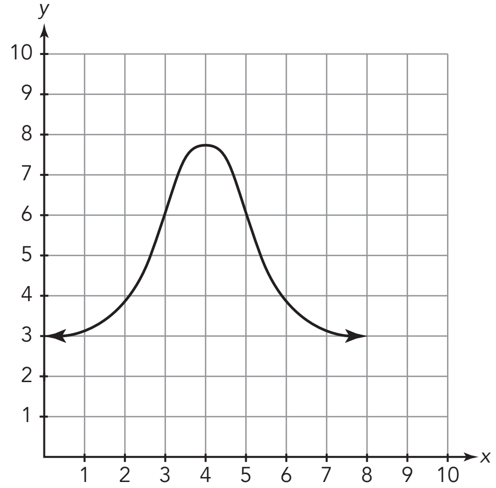
Review

Tell whether each graph is discrete or continuous. Also, tell whether each graph is increasing, decreasing, both, or neither.

1.



2.



Determine the slope and y-intercept of the linear relationship described by each equation.

3. $y = \frac{x}{2} + 5$

4. $y = \frac{x}{4}$

Calculate the slope of the line represented by each table.

5.

x	y
2	−1
3	1.5
4	4
5	6.5

6.

x	y
2	8
4	2
6	−4
9	−13