Derby Day

Slope-Intercept Form of a Line

4

$$y = mx + b$$

↑

y - intercept

WARM UP

Solve each equation for *y*.

1. $4 = \dfrac{y - 5}{3}$

2. $\dfrac{1}{2} = \dfrac{y + 3}{7}$

3. $-\dfrac{3}{4} = \dfrac{y - 17}{25}$

4. $-\dfrac{9}{5} = \dfrac{y + 31}{-8}$

LEARNING GOALS

- Write the *y*-intercept as an ordered pair.
- Determine the *y*-intercept of a linear equation from a context, a table, a graph, or an equation.
- Explain the meaning of the *y*-intercept, or initial value, when given the context of a linear equation.
- Use the slope formula to derive the slope-intercept form of a linear equation.
- Write equations of lines in slope-intercept form.
- Analyze linear relationships using slopes and initial values.

KEY TERMS

- *y*-intercept
- slope-intercept form

You have learned how to calculate the slope of a line given a graph, table, or context. How can you determine the initial value in a linear relationship from a table, equation, or graph?
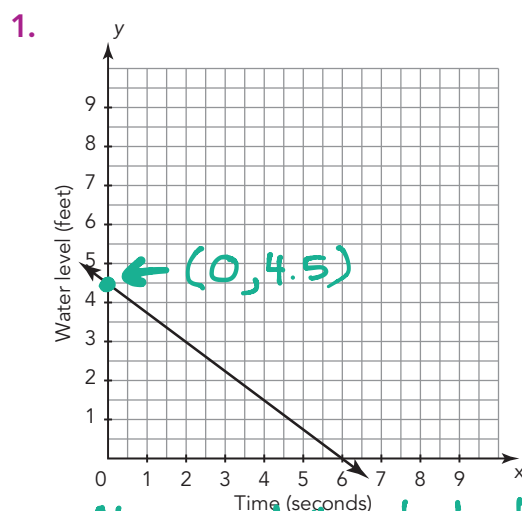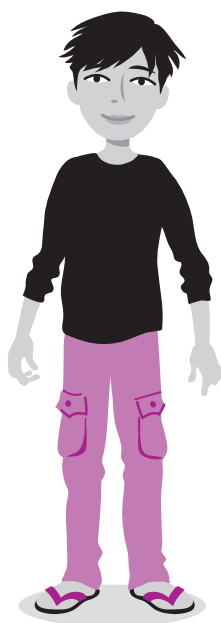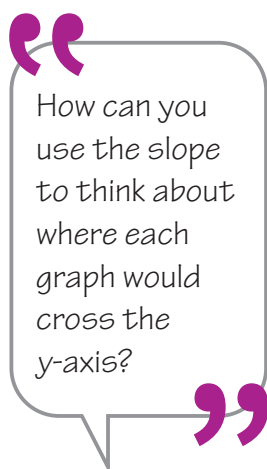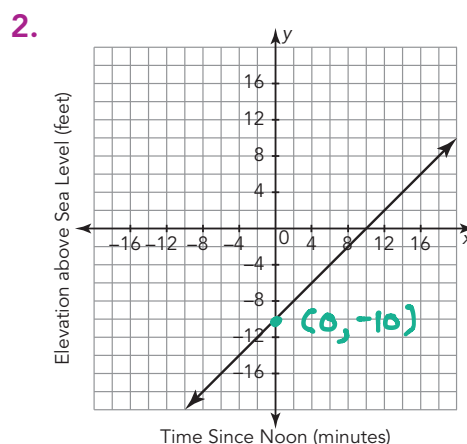
Introducing the *y*-Intercept!

The slope is one important feature of a linear equation. Another important feature is the *y-intercept*. The **y-intercept** is the *y*-coordinate of the point where a graph crosses the *y*-axis. It is the value of the dependent quantity when the independent quantity is 0. The *y*-intercept can be written as the ordered pair (0, *y*). $(0, b)$

For each graph, determine the *y*-intercept, write it as an ordered pair, and explain its meaning.

> *How can you use the slope to think about where each graph would cross the y-axis?*

1.

Water level (feet) vs. Time (seconds)

$\leftarrow (0, 4.5)$

At 0 second, the water level was at 4.5 ft.

2.

Elevation above Sea Level (feet) vs. Time Since Noon (minutes)

$(0, -10)$

At noon, the elevation was at 10 ft below sea level

3.

Distance from Home (miles) vs. Time (minutes)

$(0, 115)$
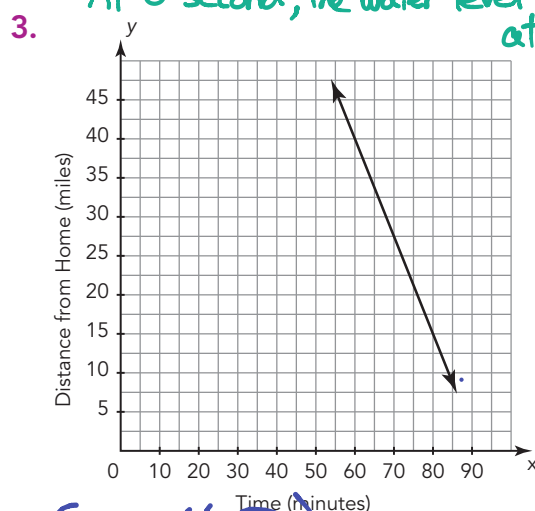
Determining the *y*-Intercept

$$y = mx + \boxed{b}$$

Just as you can determine the slope of a linear equation from a table of values or a problem situation, you can also determine the *y*-intercept. Let's start with what you already know: the slope formula.

The table of values represents a linear relationship between the variables *x* and *y*.

x	y
x_1 2	y_1 7
x_2 3	y_2 10
4	13

WORKED EXAMPLE

You can use the slope formula to determine the *y*-intercept (0, *y*) for the graph of a linear relationship.

• First, determine the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{10 - 7}{3 - 2} = \frac{3}{1} = 3$$

• Next, choose any point from the table.

(4, 13)

• Now, substitute what you know into the slope formula: *m* = 3, (4, 13), and (0, *y*).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$3 = \frac{y - 13}{0 - 4}$$

• Finally, solve for the value of the *y*-coordinate.

$$3 = \frac{y - 13}{-4}$$

$$-12 = y - 13$$

$$1 = y$$

The *y*-intercept is (0, 1).

1. **How would the worked example change if different points were chosen to calculate the slope? Explain your reasoning.**

2. **Use a different point from the table to calculate the *y*-intercept. Do you get the same *y*-intercept?**

Handwritten notes:

① Find m

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{10 - 7}{3 - 2} = 3$$

② Use the form:

$$y = mx + b$$

$$y = 3x + b$$

③ pick a point

$$(\overset{x}{2}, \overset{y}{7})$$

Substitute:

$$7 = 3(2) + b$$

$$7 = 6 + b$$
$$-6 \quad -6$$
$$1 = b$$

$$(0, 1)$$

1 $m = \dfrac{20-18}{275-250}$

$m = \dfrac{2}{25}$

Each table represents a linear relationship. Determine the *y*-intercept using the slope formula. Write the *y*-intercept in coordinate form.

2 $y = \dfrac{2}{25}x + b$

3 $(200, 14)$

$14 = \dfrac{2}{25}(200) + b$

$14 = \dfrac{400}{25} + b$

$14 = 16 + b$

$ \underset{-16}{} \underset{-16}{}$

$-2 = b \longrightarrow (0, -2)$

3.

x	y
200	14
225	16
x_1 250	y_1 18
x_2 275	y_2 20
300	22

4.

x	y
16	90
19	91
22	92
25	93
28	94

> How did you calculate the slope when given a context?

Each context represents a linear relationship. Determine the *y*-intercept using the slope formula. Write the *y*-intercept in coordinate form. Explain what the *y*-intercept represents in each problem situation.

5. Kim spent $18 to purchase a ride-all-day pass for the amusement park and to play 8 games. After playing a total of 20 games, she realized she'd spent $24.

$x = $ # of games played
$y = $ the cost

$(8, 18)(20, 24)$

$m = \dfrac{24-18}{20-8} = \dfrac{6}{12} = \dfrac{1}{2} = 0.5$

$y = 0.5x + b$

$(8, 18)$
$x y$

$18 = 0.5(8) + b$

$18 = 4 + b$

$\underset{-4}{} \underset{-4}{} \longrightarrow (0, 14)$

$14 = b$

6. Mitch saved money he received as gifts and put it toward buying a bike. When he added one week's allowance to his savings, he had $125. After 3 more weeks of saving his allowance, he had $161 toward the cost of his bike.

$x = $ # of weeks

$y = $ allowance saved

$(1, 125)(4, 161)$

$m = \dfrac{161-125}{4-1} = \dfrac{36}{3} = \$12$

$y = 12x + b$

$(1, 125)$

$125 = 12(1) + b$

$125 = 12 + b$

$\underset{-12}{} \underset{-12}{}$

$113 = b$

$(0, 113)$

(handwritten at top) $y = 12x + 113$

(handwritten) $y = mx + b$

(handwritten note, right margin)
writing Equations
① Find slope
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
② Find y-intercept (b)
- choose one pair
- substitute & solve
③ Replace m and b with numbers
$$y = \boxed{m}x + \boxed{b}$$

Now that you know how to determine the slope and y-intercept for a linear relationship from a table, graph, or context, you can use this information to write the equation of a line.

Let's use the slope and the y-intercept to determine the equation of the linear relationship represented in the table.

WORKED EXAMPLE

Just as you used the slope formula to determine the y-intercept. You can use the slope formula with an unknown point (x, y) to write an equation of the line.

- First, determine the slope and the y-intercept.

$m = 3$

y-intercept: $(0, 1)$

- Next, substitute the slope, y-intercept, and the unknown point (x, y) into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$3 = \frac{y - 1}{x - 0}$$

- Finally, solve the equation for y.

$$3 = \frac{y - 1}{x - 0}$$
$$3(x - 0) = y - 1$$
$$3x = y - 1$$
$$3x + 1 = y$$

The equation is $y = 3x + 1$.

x	y
0	1
2	7
3	10
4	13

❝ Does it matter if you substitute the y-intercept for (x_1, y_1) or for (x_2, y_2)? ❞

This linear equation is written in *slope-intercept form*. The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept. You can use this form to write linear equations when you know the slope and the y-intercept.

(handwritten) $y = mx + b$

(handwritten) $m = slope \qquad b = y\text{-intercept}$

x	y
x_1 100	y_1 10
x_2 105	y_2 6
110	2
115	−2
120	−6

By convention, the slope-intercept form is written as $y = mx + b$, but $y = b + mx$ is also correct.

1. Determine the slope, y-intercept, and the slope-intercept form of the linear equation for the relationship represented in the table.

$$m = \frac{6-10}{105-100} = \frac{-4}{5}$$

$$y = -\frac{4}{5}x + b$$
$$(2)$$
$$((10, 2)$$
$$y = mx + b$$

$$2 = -\frac{4}{5}(110) + b$$
$$2 = \frac{-440}{5} + b$$
$$2 = -88 + b \rightarrow 90 = b$$
$$+88 \quad +88 \qquad (0, 90)$$

$$\boxed{y = -\frac{4}{5}x + 90}$$

2. Write the equation for each linear relationship in slope-intercept form.

a. $m = -\frac{5}{3}$ b
   y-intercept: (0, 8)

$$y = mx + b$$
$$\boxed{y = -\frac{5}{3}x + 8}$$

b. slope: 6.2   m   b
   y-intercept: (0, −2.5)

$$\boxed{y = 6.2x - 2.5}$$

c. The line containing points (6, 19) and (0, −35)

$$m = \frac{-35-19}{0-6} = \frac{-54}{-6} = 9$$

$$x_1 \ y_1 \qquad x_2 \ y_2$$
$$y = 9x + b$$
$$-35 = 9(0) + b$$
$$-35 = b \longrightarrow (0, -35)$$

$$\boxed{y = 9x - 35}$$

d. Javi regularly checks the balance on his bus pass. Friday afternoon, his balance was $26.25. Monday morning, his balance was $1.50.

3. Consider the equations that you wrote in Question 2.

   a. Write an equation that represents a line with the same y-intercept as part (a) but a steeper slope.

   b. Write an equation that represents a line with the same y-intercept as part (b) but a steeper slope.

Each year, your class sponsors a go-kart derby to raise money for a local food bank. Jamie, a member of your class, has claimed the first-place trophy each year for the last four years. Everyone in the class is determined to capture the trophy this year.

Today is Derby Day! You and each member of your group are derby drivers competing against Jamie and Liza. Who is going to win? Your teacher will distribute Derby Day cards to your group. These cards contain the information your group needs to determine the winner.

Rules:

- The members of your group must work cooperatively to answer all the questions on the cards.

- Each member of your group will be assigned Driver A, B, C, or D.

- When you get your Driver card, do not show your card to your group members. You may only communicate the information contained on the card.

- Liza's and Jamie's cards will be shared by the entire group.

- Be sure everyone in your group discusses the entire problem and its solution.

1. **Use the graph paper located at the end of the lesson and your clue cards to help you determine the outcome of the derby.**

> Explain the rules to a partner at your table to make sure that everyone understands them.
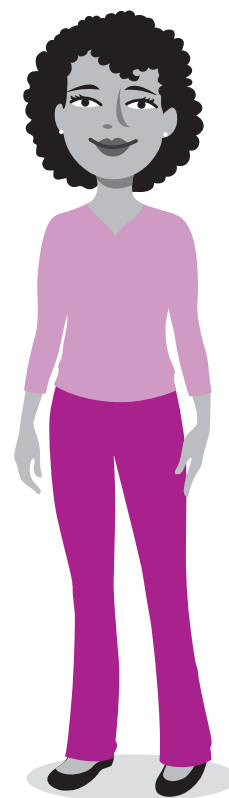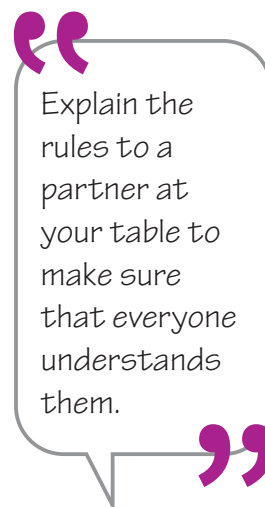
2. Use the table to organize the information from your graphs and to write equations for the drivers in slope-intercept form.

Driver	Slope	y-Intercept	Equation
A:			
B:			
C:			
D:			
Liza			
Jamie			

3. What was the speed of the driver who won the race? Explain your reasoning.

4. In what order did the drivers finish the derby? List their names or letters and the time it took them to finish.

5. After eight seconds, which driver had traveled the shortest distance from the starting line? Who had traveled the longest distance? Explain your reasoning.

6. Locate and label a point when one driver passed another driver. Describe this point and explain your reasoning.

7. Is there a point when three drivers are tied? If so, describe the point.

8. If the derby were only 20 meters long, would the order of the winners change? List their names or letters and the time it would take them to finish.

9. After 16 seconds, how far had each driver traveled from the starting line?

10. How long would the derby have to be for Driver C to win?

TALK the TALK

More or Less

Write an equation in slope-intercept form for a line with each of the given characteristics.

1. The line is decreasing from left to right and has a positive *y*-intercept.

2. The line is decreasing from left to right. The line is steeper than the line represented by the equation $y = -3x + 8$.

3. The line is increasing from left to right. The line is less steep than the line represented by the equation $y = 7x - 85$.

4. Create a context that represents a linear relationship, with (0, 22) as its *y*-intercept and a positive slope. Then write the equation of the line in slope-intercept form.

Derby Day

y

Distance (meters)

x

Time (seconds)
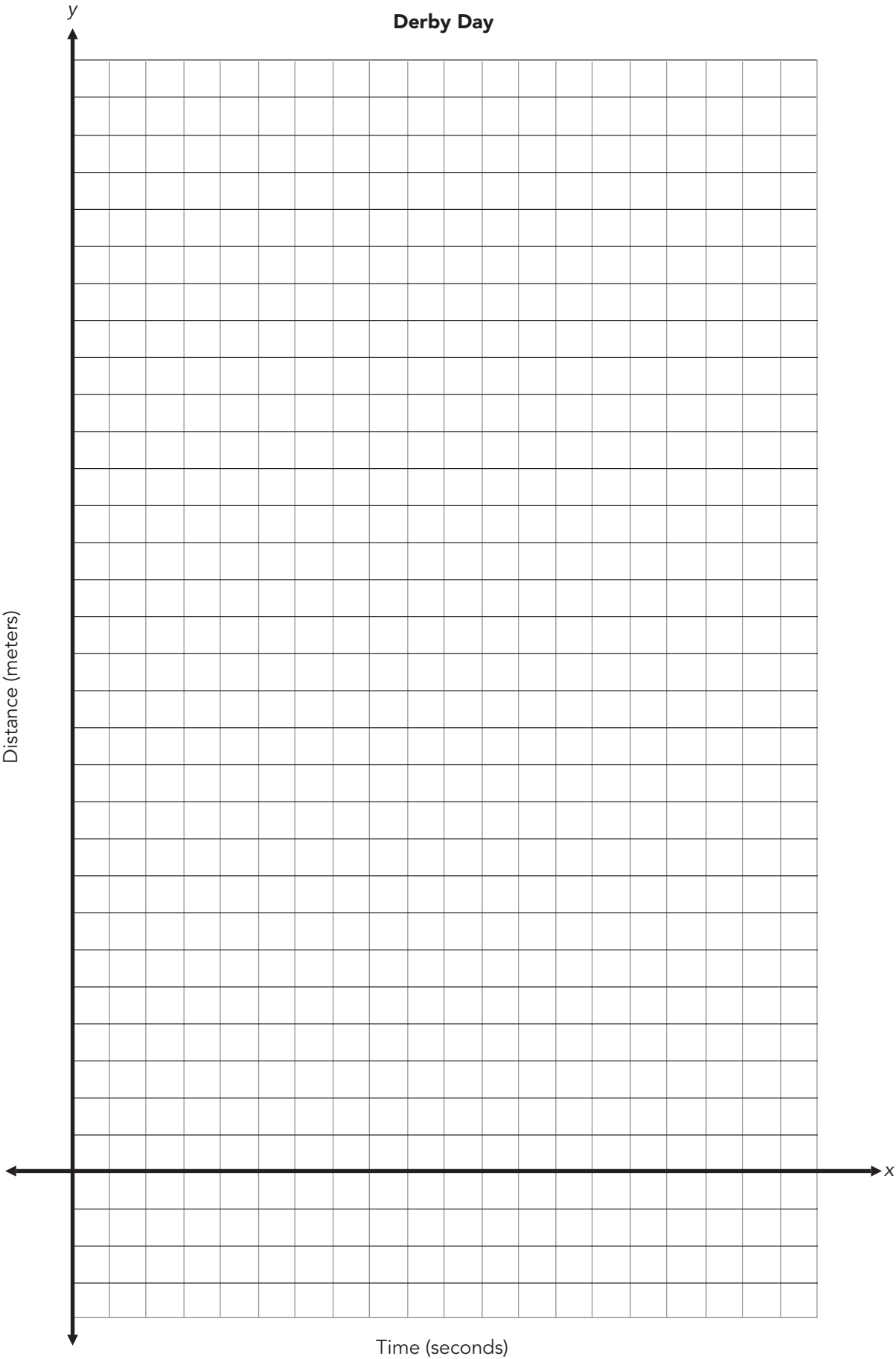
Assignment

Write

Explain how you can determine the initial value of a linear relationship, the *y*-intercept, when given two points.

Remember

The slope-intercept form of a linear equation is $y = mx + b$, where *m* is the slope of the line and $(0, b)$ is the *y*-intercept of the line.

Practice

1. Examine the linear graph. Determine the *y*-intercept and write the *y*-intercept in coordinate form. Then write the equation of the line in slope-intercept form.
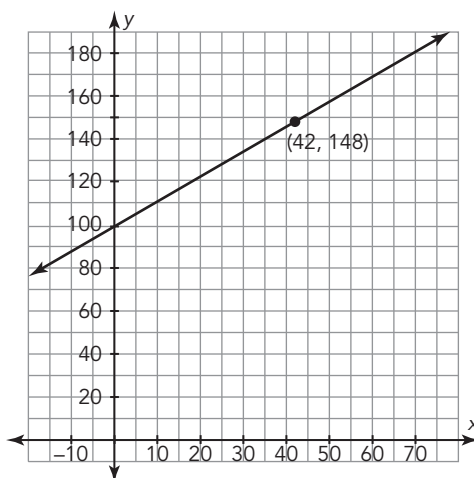
2. The table represents a linear relation. Use the table to identify the *y*-intercept. Write the *y*-intercept in coordinate form. Then write the equation in slope-intercept form.

| x	y
20	144
24	172
28	200
32	228
36	256

3. Each context represents a linear relation. Read each and determine the *y*-intercept. Write the *y*-intercept in coordinate form. Explain what the *y*-intercept represents in the problem situation. Then write the equation in slope-intercept form.

   a. The water level of a river is 34 feet, and it is receding at a rate of 0.5 foot per day.

   b. Betty worked at a golf course during the summer after eighth grade. After working for two weeks, she added her earnings to the gifts she got for graduation and found she had $570. After four more weeks of work, she had a total of $870.

4. Define the variables and write a linear equation in slope-intercept form for each problem situation. Explain the meaning of the *y*-intercept.

a. A catering company charges a fixed fee and an additional charge per person.

b. A line has a constant rate of change of $\frac{3}{7}$ and passes through the point $(0, -8)$.

c. A group bike tour costs $75 plus $12 per bike rental.

d. A salesperson receives a base salary and a percentage of the total sales for the year.

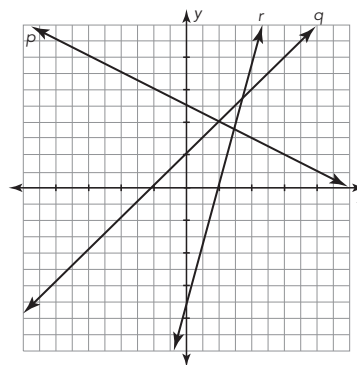| Total Sales (dollars)	Total Income (dollars)
25,000	41,250
30,000	41,500
35,000	41,750
40,000	42,000

Catering Cost
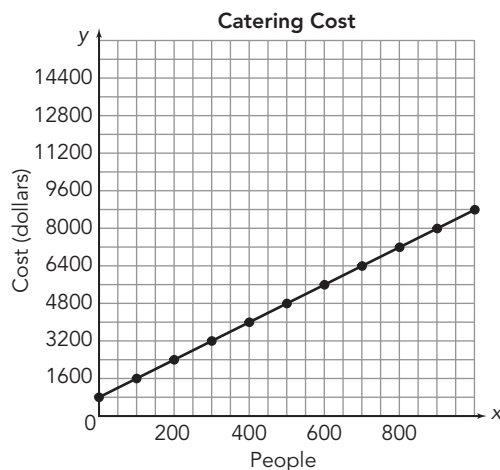
5. The graph shows three lines. The equations of the lines are as follows.

$$p: 2y = -x + 10$$
$$q: y = x + 2$$
$$r: 7x - 2y = 14$$

a. Determine the slope of each line.

b. Write the lines in order from least steep to most steep.

c. Write the equation of a line that is steeper than line *r*.

d. Write the equation of a line with a negative slope that is steeper than line *p*.

e. Write the equation of a line with a positive slope that is less steep than line *q*.

f. Write a possible context for each of the lines.

6. Draw a linear graph that is decreasing and has a *y*-intercept of $(0, 4)$. Write the equation in slope-intercept form.

7. Create a table that represents a linear relation with four values, a *y*-intercept of $(0, 6)$, and a slope of 3.

Stretch

Determine the equation for a vertical line and the equation for a horizontal line. What are the slope and *y*-intercept for each type of line?

# Review

Determine the rate of change for each situation.

1. Rosa is ordering a submarine sandwich from the corner deli. The deli charges $6.25 for a 7-inch sub. Additional toppings cost extra. Rosa's sandwich with two extra toppings costs $7.75. What is the cost per additional topping?

2. Carmen is selling pies at the cherry festival to raise money for her local volunteer fire department. She sells 85 pies for $12 each. The supplies to make the pies cost Carmen $340. What is the unit rate of the profit made for each pie?

For each graph, determine the slope and explain what the slope means in terms of the independent and dependent quantities. Then write an equation in the form $y = mx$ or $y = mx + b$ to represent the relationship between the independent and dependent quantities.

3. Kodiak is riding her skateboard down a hill, as shown in the graph.

Skateboarding
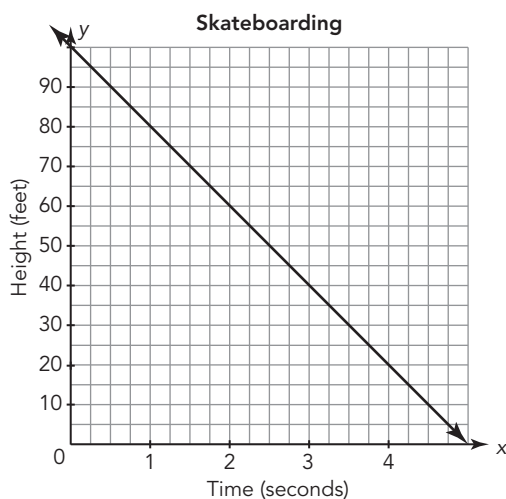
4. Andy needs a specific amount of flour to bake rolls, as shown in the graph.

Flour Needed for Rolls
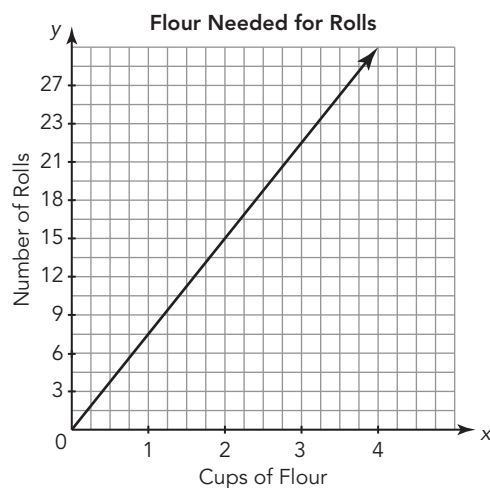
Determine the measure of each unknown angle.

5.

6.