

# Mirror, Mirror

# 4

## Reflections of Figures on the Coordinate Plane

### WARM UP

Determine each product.

1.  $-1 \times 6$

2.  $-\frac{3}{5}(-1)$

3.  $-1 \times 4.33$

4.  $4h(-1)$

### LEARNING GOALS

- Reflect geometric figures on the coordinate plane.
- Identify and describe the effect of geometric reflections on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations and reflections.

You have learned to model transformations, such as translations, rotations, and reflections. How can you model and describe these transformations on the coordinate plane?

# Getting Started

## Ambulance

The image shows the front of a typical ambulance.



1. Why does the word "ambulance" appear like this on the front?

2. Suppose you are going to replace the word *ambulance* with your name. Write your name as it appears on the front of the vehicle. How can you check that it is written correctly?

ACTIVITY  
4.1

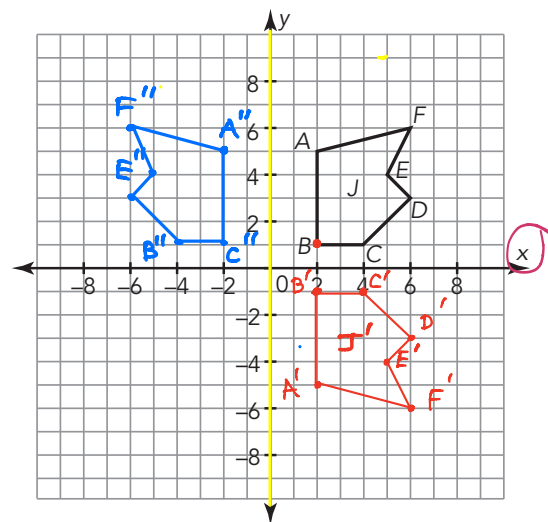
# Modeling Reflections on the Coordinate Plane



In this activity, you will reflect pre-images across the x-axis and y-axis and explore how the reflection affects the coordinates.

1. Place patty paper on the coordinate plane, trace Figure *J*, and copy the labels for the vertices on the patty paper.

a. Reflect the Figure *J* across the x-axis. Then, complete the table with the coordinates of the reflected figure.



Coordinates of <i>J</i>	Coordinates of <i>J'</i> Reflected Across x-Axis
A (2, 5)	A' (2, -5)
B (2, 1)	B' (2, -1)
C (4, 1)	C' (4, -1)
D (6, 3)	D' (6, -3)
E (5, 4)	E' (5, -4)
F (6, 6)	F' (6, -6)

b. Compare the coordinates of Figure *J'* with the coordinates of Figure *J*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

The x-coordinates are the same.

The y-coordinates are opposite.

**2. Reflect Figure J across the y-axis.**

- a. Complete the table with the coordinates of the reflected figure.

Coordinates of J	Coordinates of J'' Reflected Across y-Axis
A (2, 5)	A'' (-2, 5)
B (2, 1)	B'' (-2, 1)
C (4, 1)	C'' (-4, 1)
D (6, 3)	D'' (-6, 3)
E (5, 4)	E'' (-5, 4)
F (6, 6)	F'' (-6, 6)

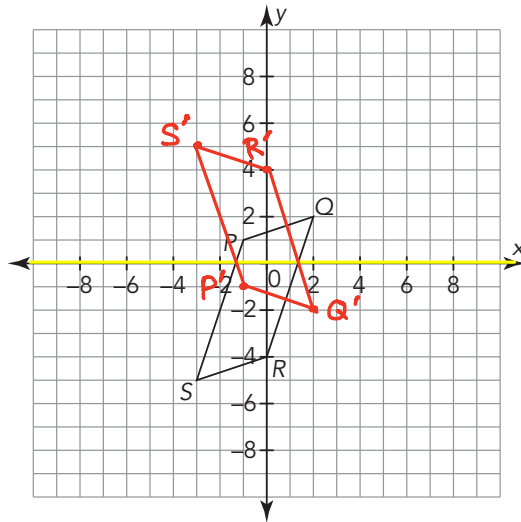
- b. Compare the coordinates of Figure J'' with the coordinates of Figure J. How are the values of the coordinates the same? How are they different? Explain your reasoning.

The y-coordinates stay the same.  
The x-coordinates are opposite.

Let's consider a new figure situated differently on the coordinate plane.

### 3. Reflect Quadrilateral $PQRS$ across the $x$ -axis.

Make a conjecture about the ordered pairs for the reflection of the quadrilateral across the  $x$ -axis.




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Make a conjecture, investigate, and then use the results to verify or justify your conjecture.

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### 4. Use patty paper to test your conjecture.

a. Complete the table with the coordinates of the reflection.

Coordinates of Quadrilateral $PQRS$	Coordinates of Quadrilateral $P'Q'R'S'$ Reflected Across the $x$ -Axis
$P(-1, 1)$	$P'(-1, -1)$
$Q(2, 2)$	$Q'(2, -2)$
$R(0, -4)$	$R'(0, 4)$
$S(-3, -5)$	$S'(-3, 5)$

b. Compare the coordinates of Quadrilateral  $P'Q'R'S'$  with the coordinates of Quadrilateral  $PQRS$ . How are the values of the coordinates the same? How are they different? Explain your reasoning.

5. Reflect Quadrilateral  $PQRS$  across the  $y$ -axis.

a. Make a conjecture about the ordered pairs for the reflection of the quadrilateral across the  $y$ -axis.

b. Use patty paper to test your conjecture. Complete the table with the coordinates of the reflection.

Coordinates of Quadrilateral $PQRS$	Coordinates of Quadrilateral $P''Q''R''S''$ Reflected Across the $y$ -Axis
$P(-1, 1)$	$P''(1, 1)$
$Q(2, 2)$	$Q''(-2, 2)$
$R(0, -4)$	$R''(0, -4)$
$S(-3, -5)$	$S''(3, -5)$

same

6. Compare the coordinates of Quadrilateral  $P''Q''R''S''$  with the coordinates of Quadrilateral  $PQRS$ . How are the values of the coordinates the same? How are they different? Explain your reasoning.

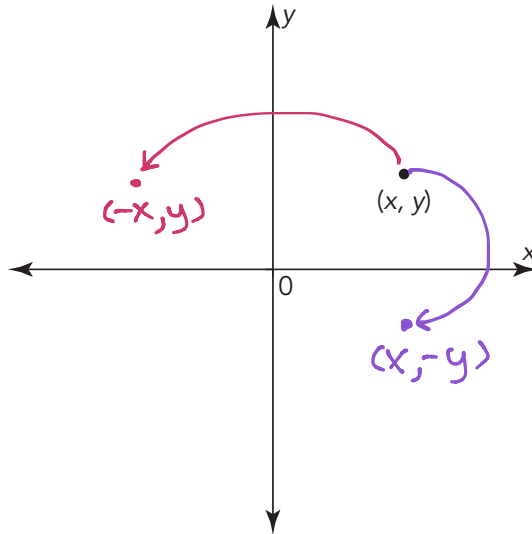
ACTIVITY  
**4.2**

# Reflecting Any Points on the Coordinate Plane



NOTES

Consider the point  $(x, y)$  located anywhere in the first quadrant.



1. Use the table to record the coordinates of each point.

- a. Reflect and graph the point  $(x, y)$  across **the x-axis** on the coordinate plane. What are the new coordinates of the reflected point in terms of  $x$  and  $y$ ?

$$(x, y) \xrightarrow{\text{x-axis}} (x, -y)$$

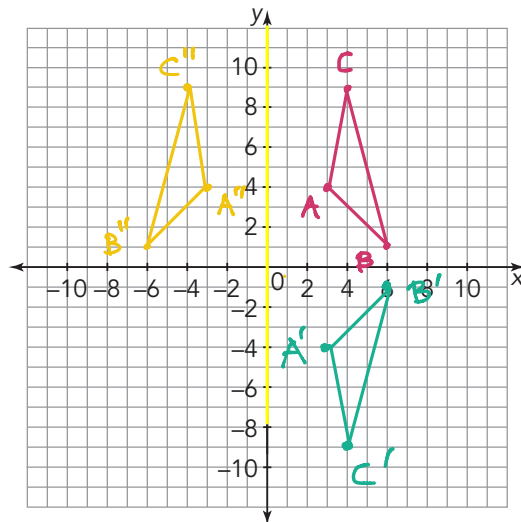
- b. Reflect and graph the point  $(x, y)$  across **the y-axis** on the coordinate plane. What are the new coordinates of the reflected point in terms of  $x$  and  $y$ ?

Original Point	Reflection Across the x-Axis	Reflection Across the y-Axis
$(x, y)$	$(x, -y)$	$(-x, y)$

$y$  changes

$x$  changes

2. Graph  $\triangle ABC$  by plotting the points  $A(3, 4)$ ,  $B(6, 1)$ , and  $C(4, 9)$ .

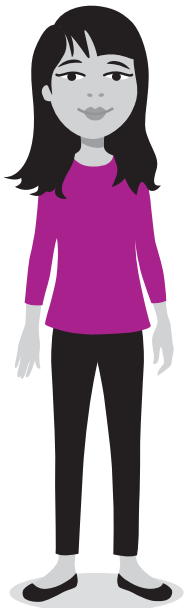


3. Use the table to record the coordinates of the vertices of each triangle.

a. Reflect  $\triangle ABC$  across the **x-axis** to form  $\triangle A'B'C'$ . Graph the triangle and then list the coordinates of the reflected triangle.



b. Reflect  $\triangle ABC$  across the **y-axis** to form  $\triangle A''B''C''$ . Graph the triangle and then list the coordinates of the reflected triangle.



Original Triangle	Triangle Reflected Across the <b>x-Axis</b>	Triangle Reflected Across the <b>y-Axis</b>
$\triangle ABC$	$\triangle A'B'C'$ <i>same</i>	$\triangle A''B''C''$ <i>same</i>
$A(3, 4)$	$A'(3, -4)$	$A''(-3, 4)$
$B(6, 1)$	$B'(6, -1)$	$B''(-6, 1)$
$C(4, 9)$	$C'(4, -9)$	$C''(-4, 9)$



Let's consider reflections of a different triangle without graphing.

4. The vertices of  $\triangle DEF$  are  $D(-7, 10)$ ,  $E(-5, 5)$ , and  $F(-1, -8)$ .

a. If  $\triangle DEF$  is reflected across the **x-axis**, what are the coordinates of the vertices of the image?

Name the triangle.

$$D'(-7, -10)$$

$$E'(-5, -5)$$

$$F'(-1, 8)$$

b. How did you determine the coordinates of the image without graphing the triangle?

keep x coordinates the same.  
change y to the opposites.

c. If  $\triangle DEF$  is reflected across the **y-axis**, what are the coordinates of the vertices of the image?

Name the triangle.

$$D''(7, 10)$$

$$E''(5, 5)$$

$$F''(1, -8)$$

d. How did you determine the coordinates of the image without graphing the triangle?

keep y coordinates the same.  
change x to the opposites.

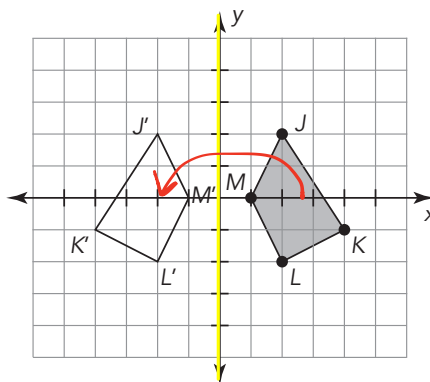
ACTIVITY  
4.3

# Verifying Congruence Using Reflections and Translations



Just as with translations, one way to verify that two figures are congruent is to show that the same sequence of reflections moves all the points of one figure onto all the points of the other figure.

1. Consider the two figures shown.



a. Complete the table with the corresponding coordinates of each figure.

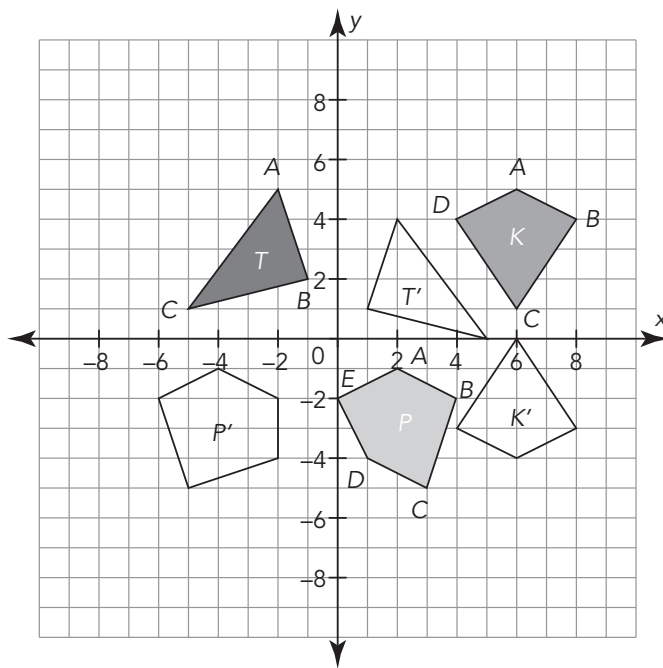
Coordinates of JKLM	Coordinates of J'K'L'M'
J (2, 2)	J' (-2, 2)
K (4, -1)	K' (-4, -1)
L (2, -2)	L' (-2, -2)
M (1, 0)	M' (-1, 0)

\* reflection over y-axis

b. Is Quadrilateral JKLM congruent to Quadrilateral J'K'L'M'? Describe the sequence of rigid motions to verify your conclusion.

Remember, a rigid motion is a transformation that preserves the size and shape of the figure.

2. Study the figures shown on the coordinate plane.



Determine whether each pair of figures are congruent. Then describe the sequence of rigid motions to verify your conclusion.

↳ translation, reflection

a. Is Figure K congruent to Figure K'?

Yes, translate one down then reflect over x-axis

b. Is Figure P congruent to Figure P'?

No

c. Is Figure T congruent to Figure T'?

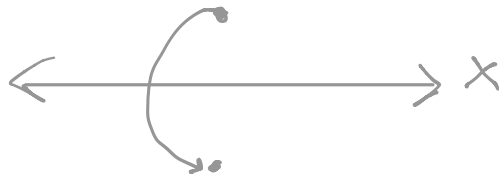
Yes, translate one down then reflect over y-axis

TALK the TALK 

## Reflecting on Reflections

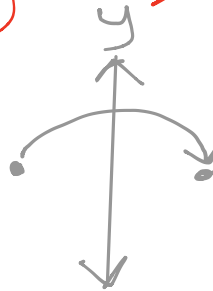
1. Describe how the ordered pair  $(x, y)$  of any figure changes when the figure is reflected across the **x-axis**.

$$(x, y) \longrightarrow (x, -y)$$



2. Describe how the ordered pair  $(x, y)$  of any figure changes when the figure is reflected across the **y-axis**.

$$(x, y) \longrightarrow (-x, y)$$



# Assignment

## Write

In your own words, explain how reflections across the  $x$ -axis and across the  $y$ -axis each affect the coordinates of the points of a figure.

## Remember

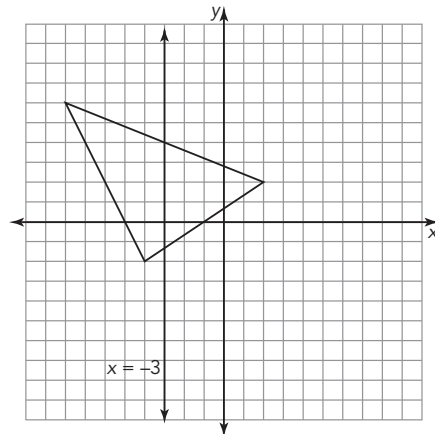
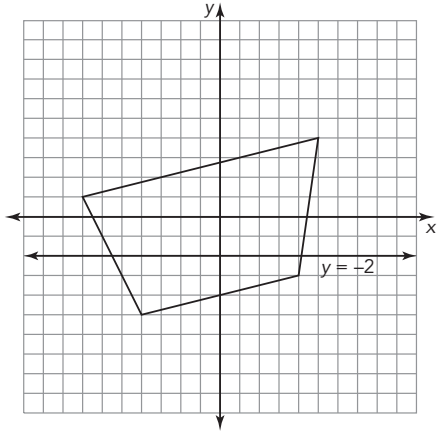
A reflection “flips” a figure across a line of reflection. A reflection is a rigid motion that preserves the size and shape of figures.

## Practice

1. Use a coordinate plane to complete parts (a) through (i).
  - a. Plot the points  $(0, 0)$ ,  $(-7, 5)$ ,  $(-7, 8)$ ,  $(-4, 8)$  and connect them with straight lines in the order in which they are given. Connect the last point to the first point to complete the figure. Label it 1.
  - b. List the ordered pairs of Quadrilateral 1 if it is reflected across the  $y$ -axis. Explain how you can determine the ordered pairs of the reflection without graphing it. Plot the reflection described. Label it 2.
  - c. List the ordered pairs of Quadrilateral 2 if it is reflected over the  $x$ -axis. Explain how you can determine the ordered pairs of the reflection without graphing it. Plot the reflection described. Label it 3.
  - d. List the ordered pairs of Quadrilateral 1 if it is reflected over the  $x$ -axis. Explain how you can determine the ordered pairs of the reflection without graphing it. Plot the reflection described. Label it 4.
2. Write a general statement about how to determine the ordered pairs of the vertices of a figure if it is reflected across the  $x$ -axis.
3. Write a general statement about how to determine the ordered pairs of the vertices of a figure if it is reflected across the  $y$ -axis.

## Stretch

1. Reflect the quadrilateral across the line  $y = -2$ .
2. Reflect the triangle across the line  $x = -3$ .



## Review

Determine the coordinates of the image following each given translation.

1. Triangle  $ABC$  with coordinates  $A(2, 4)$ ,  $B(3, 6)$ , and  $C(5, 1)$  is translated 4 units horizontally.
2. Parallelogram  $DEFG$  with coordinates  $D(0, 2)$ ,  $E(1, 5)$ ,  $F(6, 5)$ , and  $G(5, 2)$  is translated  $-7$  units horizontally.
3. For each translation described, what is the relationship between the image and pre-image?

Calculate each product or quotient.

4.  $\frac{-24.6}{-6}$
5.  $4.3(-2.1)$