Half Turns and Quarter Turns

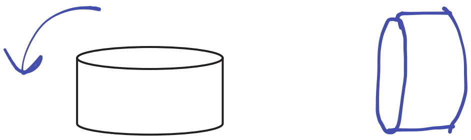
5

Rotations of Figures on the Coordinate Plane

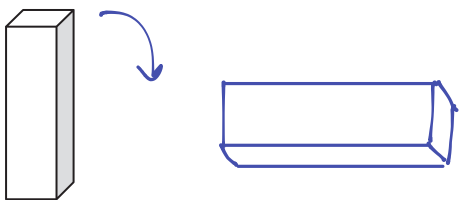
WARM UP

1. Redraw each given figure as described.
   a. so that it is turned 180° clockwise
   Before: After:

   b. so that it is turned 90° counterclockwise
   Before: After:



   c. so that it is turned 90° clockwise
   Before: After:



LEARNING GOALS

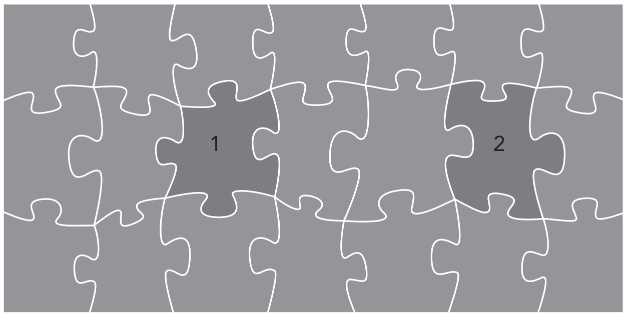
- Rotate geometric figures on the coordinate plane 90° and 180°.
- Identify and describe the effect of geometric rotations of 90° and 180° on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations, reflections, and rotations.

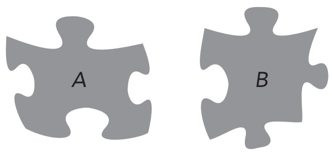
You have learned to model rigid motions, such as translations, rotations, and reflections. How can you model and describe these transformations on the coordinate plane?

Getting Started

Jigsaw Transformations

There are just two pieces left to complete this jigsaw puzzle.





1. **Which puzzle piece fills the missing spot at 1? Describe the translations, reflections, and rotations needed to move the piece into the spot.**

B rotate 180°
slide up and left

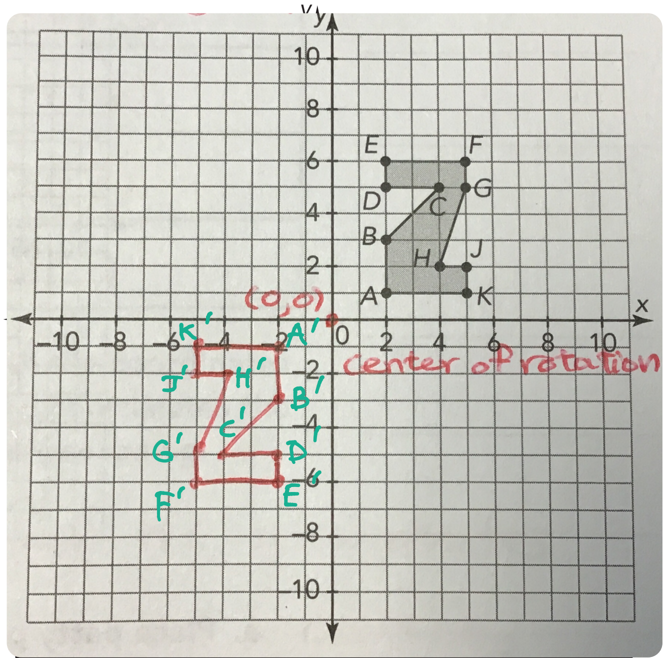
2. **Which puzzle piece fills the missing spot at 2? Describe the translations, reflections, and rotations needed to move the piece into the spot.**

A rotate 90° clockwise
slide up and right

ACTIVITY 5.1

Modeling Rotations on the Coordinate Plane

In this activity, you will investigate rotating pre-images to understand how the rotation affects the coordinates of the image.



1. Rotate the figure 180° about the origin.

a. Place patty paper on the coordinate plane, trace the figure, and copy the labels for the vertices on the patty paper.

b. Mark the origin, (0, 0), as the center of rotation. Trace a ray from the origin on the *x*-axis. This ray will track the angle of rotation.

c. Rotate the figure 180° about the center of rotation. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane. Finally, complete the table with the coordinates of the rotated figure.

Coordinates of Pre-Image	Coordinates of Image
A (2, 1)	A′(−2,−1)
B (2, 3)	B′(−2,−3)
C (4, 5)	C′(−4,−5)
D (2, 5)	D′(−2,−5)
E (2, 6)	E′(−2,−6)
F (5, 6)	F′(−5,−6)
G (5, 5)	G′(−5,−5)
H (4, 2)	H′(−4,−2)
J (5, 2)	J′(−5,−2)
K (5, 1)	K′(−5,−1)

d. Compare the coordinates of the rotated figure with the coordinates of the original figure. How are the values of the coordinates the same? How are they different? Explain your reasoning.

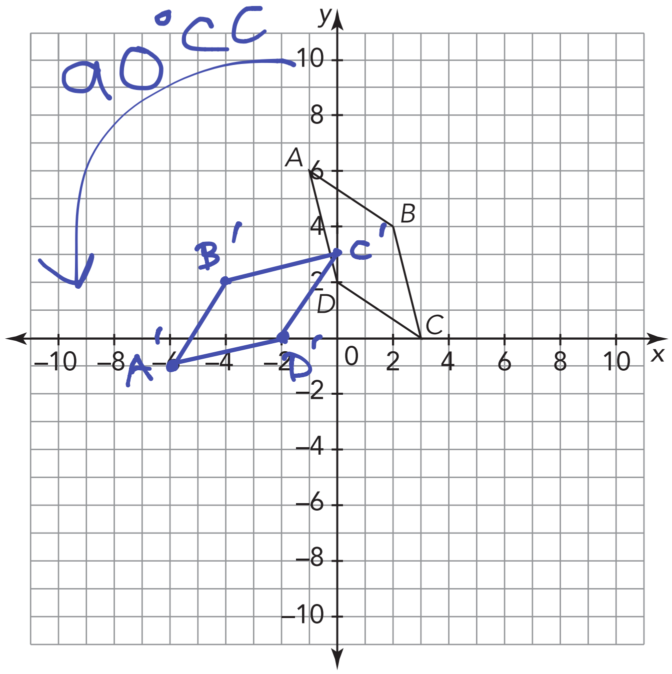
Both coordinates are opposite.

$(x, y) \xrightarrow{180^\circ} (-x, -y)$

NOTES

Now, let's investigate rotating a figure 90° about the origin.

2. Consider the parallelogram shown on the coordinate plane.



a. Place patty paper on the coordinate plane, trace the parallelogram, and then copy the labels for the vertices.

CC

b. Rotate the figure 90° counterclockwise about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.

c. Complete the table with the coordinates of the pre-image and the image.

Coordinates of Pre-Image	Coordinates of Image
A (-1, 6)	A' (-6, -1)
B (2, 4)	B' (-4, 2)
C (3, 0)	C' (0, 3)
D (0, 2)	D' (-2, 0)

d. Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.

The x and y coordinates switched.

The new x-coordinates are opposite

3. Make conjectures about how a counterclockwise 90° rotation and a 180° rotation affect the coordinates of any point (x, y).

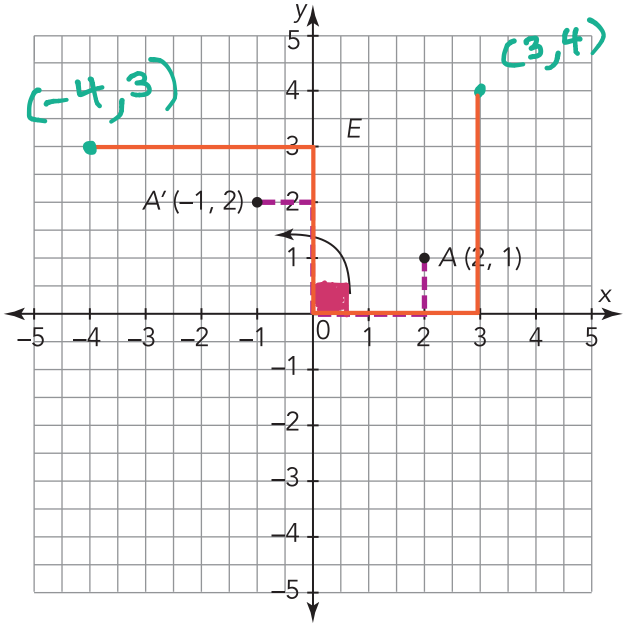
$$(x, y) \xrightarrow{180^\circ} (-x, -y)$$

$$(x, y) \xrightarrow{90^\circ\ cc} (-y, x)$$

You can use steps to help you rotate geometric objects on the coordinate plane.

Let's rotate a point 90° counterclockwise about the origin.

Step 1: Draw a "hook" from the origin to point *A*, using the coordinates and horizontal and vertical line segments as shown.



Step 2: Rotate the "hook" 90° counterclockwise as shown.

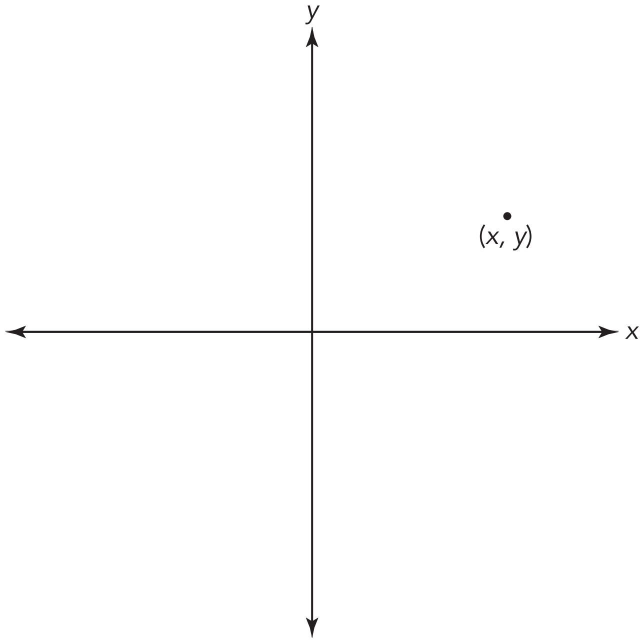
Point *A*′ is located at (−1, 2). Point *A* has been rotated 90° counterclockwise about the origin.

4. What do you notice about the coordinates of the rotated point? How does this compare with your conjecture?

ACTIVITY 5.2

Rotating Any Points on the Coordinate Plane

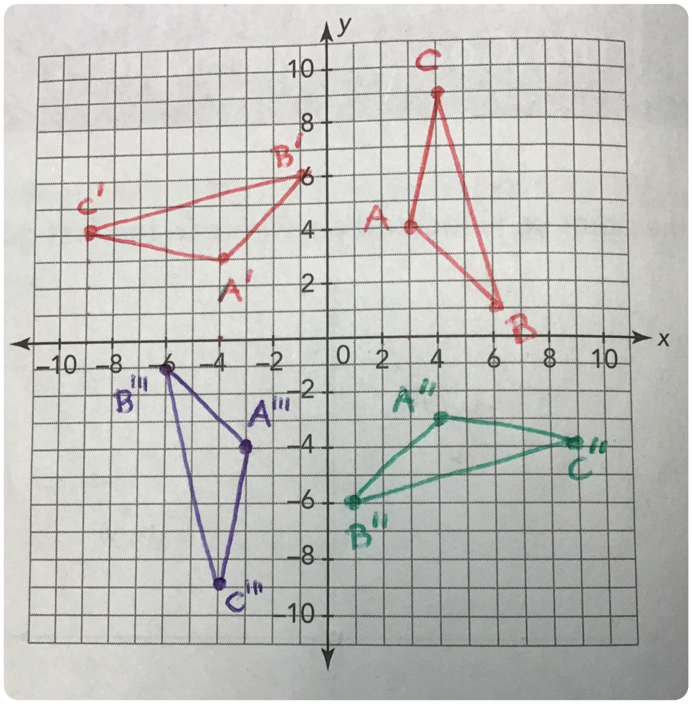
Consider the point (x, y) located anywhere in the first quadrant.



1. Use the origin, (0, 0), as the point of rotation. Rotate the point (x, y) as described in the table and plot and label the new point. Then record the coordinates of each rotated point in terms of x and y.

Original Point	Rotation About the Origin 90° Counterclockwise	Rotation About the Origin 90° Clockwise	Rotation About the Origin 180°
(x, y)	$(-y, x)$	$(y, -x)$	$(-x, -y)$
(-1,2)	(-2,-1)	(2, 1)	(1,-2)

2. Graph ΔABC by plotting the points A (3, 4), B (6, 1), and C (4, 9).



Use the origin, (0, 0), as the point of rotation. Rotate ΔABC as described in the table, graph and label the new triangle. Then record the coordinates of the vertices of each triangle in the table.

Original Triangle	Rotation About the Origin 90° Counterclockwise (−y, x)	Rotation About the Origin 90° Clockwise (y, −x)	Rotation About the Origin 180° (−x, −y)
ΔABC	$\Delta A'B'C'$	$\Delta A''B''C''$	$\Delta A'''B'''C'''$
A (3, 4)	(−4, 3)	(4, −3)	A'''(−3, −4)
B (6, 1)	(−1, 6)	(1, −6)	B'''(−6, −1
C (4, 9)	(−9, 4)	(9, −4)	C'''(−4, −9)

NOTES

Let's consider rotations of a different triangle without graphing.

3. The vertices of ΔDEF are D (−7, 10), E (−5, 5), and F (−1, −8).

a. If ΔDEF is rotated 90° counterclockwise about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

$(-y, x)$

$D(-7, 10) \rightarrow D'(-10, -7)$

$E(-5, 5) \rightarrow E'(-5, -5)$

$F(-1, -8) \rightarrow F'(8, -1)$

b. How did you determine the coordinates of the image without graphing the triangle?

$(y, -x)$

c. If ΔDEF is rotated 90° clockwise about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

$D(-7, 10) \rightarrow (10, 7)$

$E(-5, 5) \rightarrow (5, 5)$

$F(-1, -8) \rightarrow (-8, 1)$

NOTES

d. How did you determine the coordinates of the image without graphing the triangle?

$(-x, -y)$

e. If ΔDEF is rotated 180° about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

$D(-7, 10) \rightarrow (7, -10)$

$E(-5, 5) \rightarrow (5, -5)$

$F(-1, -8) \rightarrow (1, 8)$

f. How did you determine the coordinates of the image without graphing the triangle?

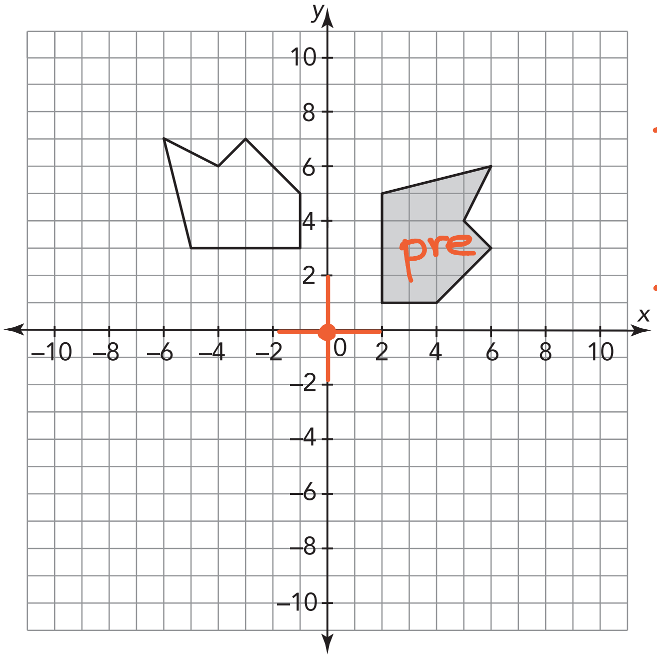
ACTIVITY
5.3

Verifying Congruence Using Rigid Motions

more than one transformations.

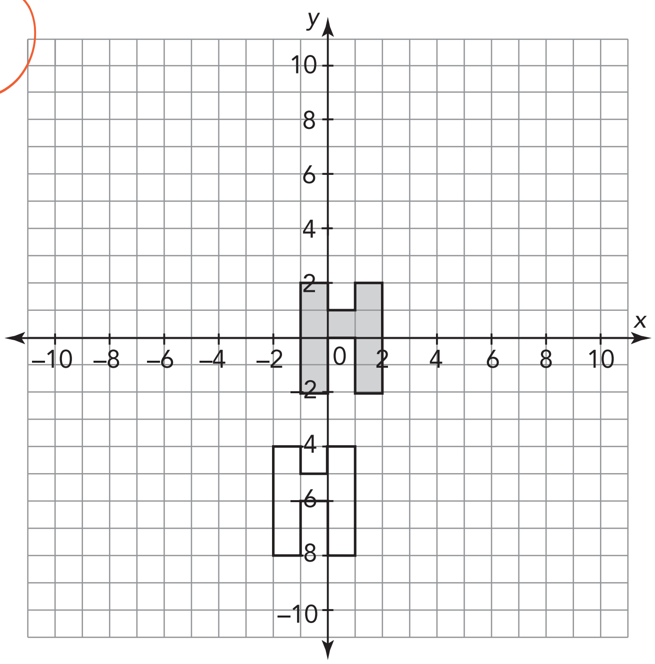
Describe a sequence of rigid motions that can be used to verify that the shaded pre-image is congruent to the image.

1.



- Rotate 90° counterclockwise
- translate up 1

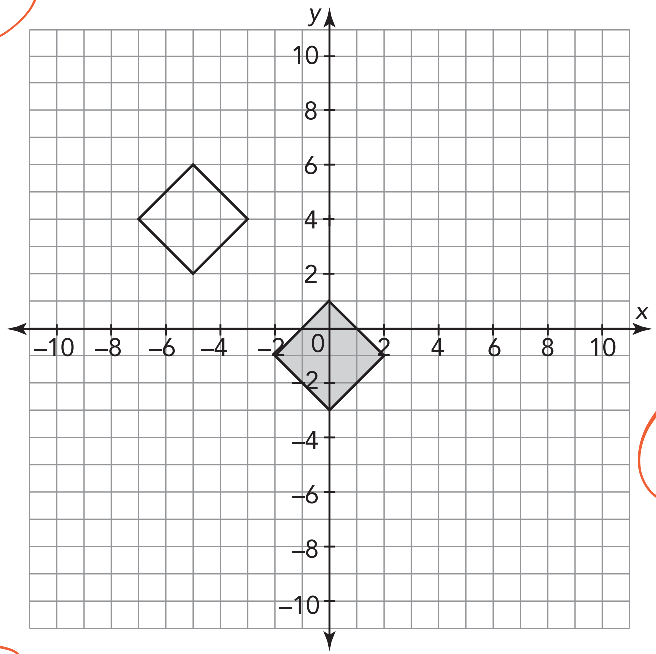
2.



Translate left 1, down 6

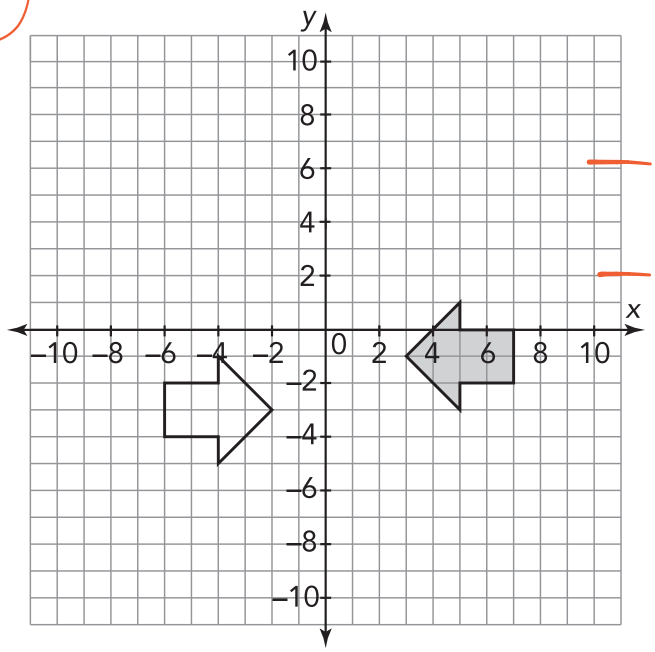
$(x-1, y-6)$

3.



Translat
5 left
5 up
(x−5, y+5)

4.



— Rotate 180°
— Translate
right 1
down 4

NOTES

TALK the TALK

Just the Coordinates

Using what you know about rigid motions, verify that the figures represented by the coordinates are congruent. Describe the sequence of rigid motions to explain your reasoning.

1. $\triangle QRS$ has coordinates Q (1, −1), R (3, −2), and S (2, −3). $\triangle Q'R'S'$ has coordinates Q' (5, −4), R' (6, −2), and S' (7, −3).

2. Rectangle $MNPQ$ has coordinates M (3, −2), N (5, −2), P (5, −6), and Q (3, −6). Rectangle $M'N'P'Q'$ has coordinates M' (0, 0), N' (−2, 0), P' (−2, 4), and Q' (0, 4).

Assignment

Write

In your own words, explain how each rotation about the origin affects the coordinate points of a figure.

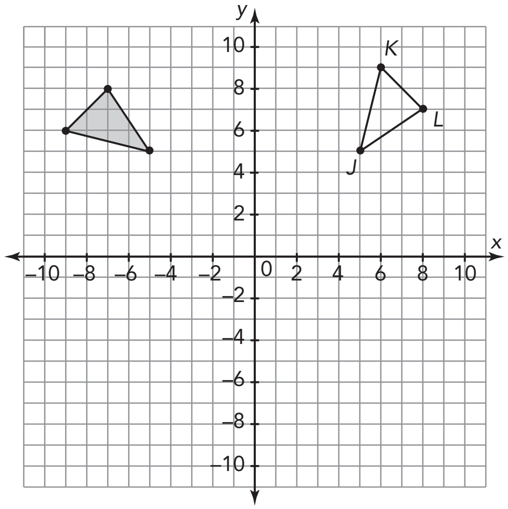
a. a counterclockwise rotation of 90°
b. a clockwise rotation of 90°
c. a rotation of 180°

Remember

A rotation "turns" a figure about a point. A rotation is a rigid motion that preserves the size and shape of figures.

Practice

1. Use △*JKL* and the coordinate plane to answer each question.

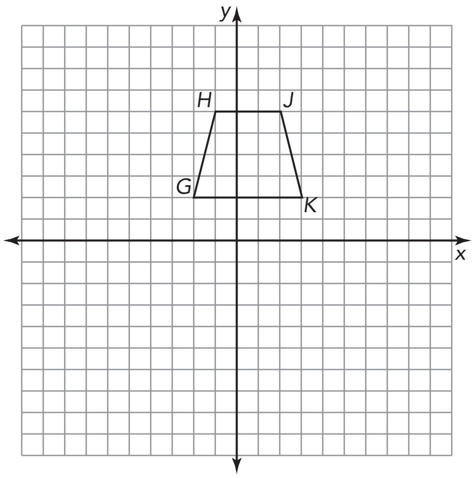


a. List the coordinates of each vertex of △*JKL*.
b. Describe the rotation that you can use to move △*JKL* onto the shaded area on the coordinate plane. Use the origin as the point of rotation.
c. Determine what the coordinates of the vertices of the rotated △*J′K′L′* will be if you perform the rotation you described in your answer to part (b). Explain how you determined your answers.
d. Verify your answers by graphing △*J′K′L′* on the coordinate plane.

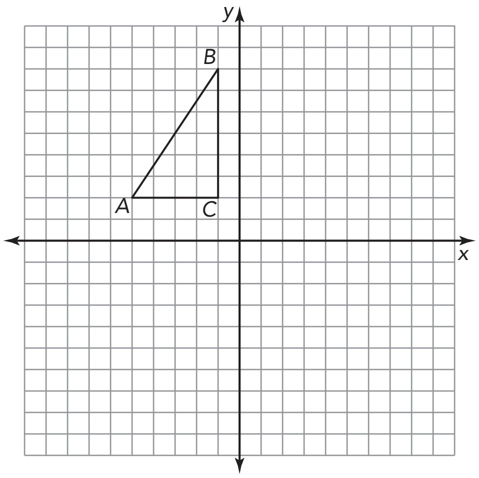
2. Determine the coordinates of each triangle's image after the given transformation.
   a. Triangle *ABC* with coordinates *A* (3, 4), *B* (7, 7), and *C* (8, 1) is translated 6 units left and 7 units down.
   b. Triangle *DEF* with coordinates *D* (−2, 2), *E* (1, 5), and *F* (4, −1) is rotated 90° counterclockwise about the origin.
   c. Triangle *GHJ* with coordinates *G* (2, −9), *H* (3, 8), and *J* (1, 6) is reflected across the *x*-axis.
   d. Triangle *KLM* with coordinates *K* (−4, 2), *L* (−8, 7), and *M* (3, −3) is translated 4 units right and 9 units up.
   e. Triangle *NPQ* with coordinates *N* (12, −3), *P* (1, 2), and *Q* (9, 0) is rotated 180° about the origin.

Stretch

1. Rotate Trapezoid *GHJK* 90° clockwise around point *G*.



2. Rotate $\triangle ABC$ 135° clockwise around point *C*.



Review

Given a triangle with the vertices *A* (1, 3), *B* (4, 8), and *C* (5, 2). Determine the vertices of each described transformation.

1. A reflection across the *x*-axis.
2. A reflection across the *y*-axis.
3. A translation 5 units horizontally.
4. A translation −4 units vertically.

Rewrite each expression using properties.

5. $2(x + 4) - 3(x - 5)$
6. $10 - 8(2x - 7)$