

Solving Linear Equations NOTES

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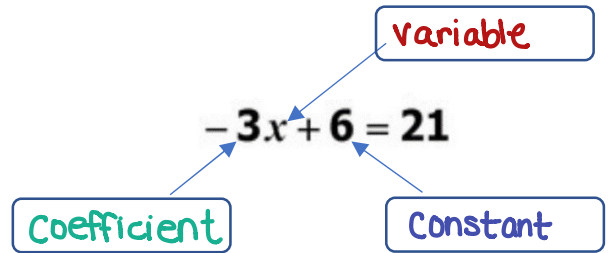
I can solve one and two-step equations

vocabulary

When solving equations, you will need to use inverse, or opposite operations to isolate the variable.

Inverses:

- Addition is subtraction
- Subtraction is addition
- Multiplication is division
- Division is multiplication



One-Step Equations

1.
$$\begin{array}{r} p + 12 = 10 \\ -12 \quad -12 \\ \hline p = -2 \end{array}$$

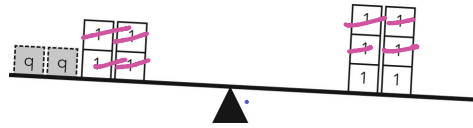
2.
$$\begin{array}{r} 5 = m - 8 \\ +8 \quad +8 \\ \hline 13 = m \end{array}$$

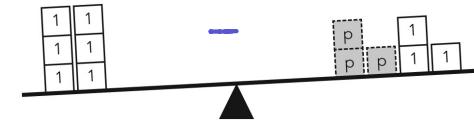
3.
$$\begin{array}{r} -3a = 12 \\ -3 \quad -3 \\ \hline a = -4 \end{array}$$


4.
$$\begin{array}{r} \frac{t}{3} = -6 \quad (3) \\ \times 3 \quad \times 3 \\ \hline t = -18 \end{array}$$

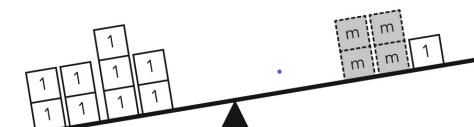
Two-Step Equations

When solving equations, remember it's all about keeping BALANCE

5. 
q = 1
$$\begin{array}{r} 2q + 4 = 6 \\ -4 \quad -4 \\ \hline 2q = 2 \\ \frac{2q}{2} = \frac{2}{2} \\ q = 1 \end{array}$$

6. 
p = 1
$$\begin{array}{r} 6 = 3p + 3 \\ -3 \quad -3 \\ \hline 3 = 3p \\ \frac{3}{3} = \frac{3p}{3} \\ 1 = p \end{array}$$

7. 
a = 3
$$\begin{array}{r} 3a + 1 = 10 \\ -1 \quad -1 \\ \hline 3a = 9 \\ \frac{3a}{3} = \frac{9}{3} \\ a = 3 \end{array}$$

8. 
m = 2
$$\begin{array}{r} 9 = 4m + 1 \\ -1 \quad -1 \\ \hline 8 = 4m \\ \frac{8}{4} = \frac{4m}{4} \\ m = 2 \end{array}$$

Two-Step Equations

To solve a Two-Step Equation:

1. Draw a line through the equal sign to show balance.
2. Undo the Addition/Subtraction (to remove the constant term)
3. Undo the Multiplication/Division (to remove the coefficient)

$$\begin{array}{r} 4x - 8 = 16 \\ \hline +8 \quad +8 \\ \hline 4x = 24 \\ \hline \frac{4x}{4} = \frac{24}{4} \end{array}$$

$$x = 6$$

$$\begin{array}{r} \frac{y}{12} - 5 = 11 \\ \hline +5 \quad +5 \\ \hline \frac{y}{12} = 16 \quad (12) \\ \hline y = 192 \end{array}$$

$$y = 192$$

$$\begin{array}{r} -61 = 7y - 26 \\ \hline +26 \quad +26 \\ \hline -35 = 7y \\ \hline \frac{-35}{7} = \frac{7y}{7} \end{array}$$

$$x = 5$$

$$\begin{array}{r} 4 - 3n = 43 \\ \hline -4 \quad -4 \\ \hline -3n = 39 \\ \hline \frac{-3n}{-3} = \frac{39}{-3} \end{array}$$

$$x = -13$$

$$\begin{array}{r} \frac{x}{3} + 5 = -4 \\ \hline -5 \quad -5 \\ \hline \frac{x}{3} = -9 \quad (3) \\ \hline x = -27 \end{array}$$

$$x = -27$$

$$\begin{array}{r} 23 - x = 13 \\ \hline -23 \quad -23 \\ \hline -x = -10 \\ \hline \frac{-x}{-1} = \frac{-10}{-1} \end{array}$$

$$x = 10$$

$$\begin{array}{r} 3x + 6 = -18 \\ \hline -6 \quad -6 \\ \hline 3x = -24 \\ \hline \frac{3x}{3} = \frac{-24}{3} \end{array}$$

$$x = -8$$

$$\begin{array}{r} 12 = -2x + 10 \\ \hline -10 \quad -10 \\ \hline 2 = -2x \\ \hline \frac{2}{-2} = \frac{-2x}{-2} \end{array}$$

$$x = -1$$

$$\begin{array}{r} 14 = 6 - 2x \\ \hline -6 \quad -6 \\ \hline 8 = -2x \\ \hline \frac{8}{-2} = \frac{-2x}{-2} \end{array}$$

$$x = -4$$

$$\begin{array}{r} 14 = 3 - x \\ \hline -3 \quad -3 \\ \hline 11 = -x \\ \hline \frac{11}{-1} = \frac{-x}{-1} \end{array}$$

$$x = -11$$

$$\begin{array}{r} \frac{x}{4} + 10 = 1 \\ \hline -10 \quad -10 \\ \hline \frac{x}{4} = -9 \quad (4) \\ \hline x = -36 \end{array}$$

$$x = -36$$

$$\begin{array}{r} (2) \frac{-x}{2} = -6 \quad (2) \\ \hline -x = -12 \\ \hline \frac{-x}{-1} = \frac{-12}{-1} \end{array}$$

$$x = 12$$

I can solve multi-step equations with variables on one side of the equation.

Combining Like Terms

(see Slide-Share presentation)

“Like terms” are terms that contain the same letter variables which are raised to the same exact powers. Only the first number “coefficients” of the terms are different.

Example:

$3h$ and $-h$ **YES** – letters the same ($-h = -1h$)

$5p^2q^3$ and $-4p^2q^3$ **YES** – letters & powers same

Non-Example

$4g$ and $4h$ **NO** – letter variables are different.

$2x^2y^3$ and $2x^2y^5$ **NO** – y powers are different.

Consider the following take-away meal:



Write an equation to show your meal order, and then combine like terms.

$$2b + 1f + 1d + 3b + 2f + 2d = 5b + 3f + 3d$$

$$7x + 2x - 5 + x - 2x + 9 = 8x + 4$$

$$x - 5 - 9x - 3 = -8x - 8$$

Solve for the variable in each of the following equations

$$7x + 2x - 5 + x - 2x + 9 = 45$$

$$8x + 4 = 45$$

$$\frac{8x}{8} = \frac{41}{8} \quad \boxed{x = \frac{41}{8}}$$

$$x - 5 - 9x - 3 = -48$$

$$\frac{-8x - 8}{-8} = \frac{-40}{-8} \quad \boxed{x = 5}$$

$$12 = -2x + 10 + 8x - 10$$

$$12 = 6x$$

$$2 = x$$

$$9x + 12 - 2x - 5 + 7x = 21$$

$$\frac{14x + 7}{14} = \frac{21}{14}$$

$$\frac{14x}{14} = \frac{-28}{14} \quad \boxed{x = -2}$$

Distributive Property

You can use the distributive property to simplify expressions. To distribute, multiply the term on the outside of the parentheses to both terms on the inside of parentheses.

$$4(x + 2)$$

$$4x + 8$$

$$3(x - 5)$$

$$3x - 15$$

$$-7(2x - 5)$$

$$-14x + 35$$

$$8(2x - 5)$$

$$16x - 40$$



Multi-Step Equations with variables on ONE SIDE

$$\begin{aligned}
 2(5 - x) &= 9 \\
 10 - 2x &= 9 \\
 -10 & \quad -10 \\
 \hline
 -2x &= -1 \\
 -2 & \quad -2 \\
 \hline
 x &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 3x - 7x - 8 &= 24 \\
 -4x - 8 &= 24 \\
 +8 & \quad +8 \\
 \hline
 -4x &= 32 \\
 -4 & \quad -4 \\
 \hline
 x &= -8
 \end{aligned}$$

$$\begin{aligned}
 12 &= -2x - (x + 3) \\
 12 &= -2x - x - 3 \\
 12 &= -3x - 3 \\
 \frac{15}{-3} &= \frac{-3x}{-3} \\
 x &= -5
 \end{aligned}$$

$$\begin{aligned}
 8(2x - 5) - 9x &= -33 \\
 16x - 40 - 9x &= -33 \\
 8x - 40 &= -33 \\
 8x &= 7 \\
 x &= \frac{7}{8}
 \end{aligned}$$

$$\begin{aligned}
 2(3x + 5) - 4 &= 18 \\
 6x + 10 - 4 &= 18 \\
 6x + 6 &= 18 \\
 6x &= 12 \\
 x &= 2
 \end{aligned}$$

$$\begin{aligned}
 13 &= 12x - 5 - 3x \\
 13 &= 9x - 5 \\
 18 &= 9x \\
 2 &= x
 \end{aligned}$$

$$\begin{aligned}
 x + 4(x + 3) &= 17 \\
 x + 4x + 12 &= 17 \\
 5x + 12 &= 17 \\
 5x &= 5 \\
 x &= 1
 \end{aligned}$$

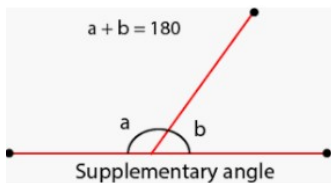
$$\begin{aligned}
 12x - (4x + 10) &= 54 \\
 12x - 4x - 10 &= 54 \\
 8x - 10 &= 54 \\
 8x &= 64 \\
 x &= 8
 \end{aligned}$$

$$\begin{aligned}
 5(2 + x) + 5(3 + 7x) &= 25 \\
 10 + 5x + 15 + 35x &= 25 \\
 25 + 40x &= 25 \\
 40x &= 0 \\
 x &= 0
 \end{aligned}$$

$$\begin{aligned}
 2(-7x + 5) + 2 + 12x &= 3 \\
 -14x + 10 + 2 + 12x &= 3 \\
 -2x + 12 &= 3 \\
 -2x &= -9 \\
 x &= \frac{9}{2}
 \end{aligned}$$

Solve for x and then find the measure of each missing angle.

Application to Supplementary Angles



$$\begin{aligned}
 &\leftarrow \begin{array}{c} 5x^\circ \\ 5(21) \\ 105^\circ \end{array} \begin{array}{c} (3x + 12)^\circ \\ 3(21) + 12 \\ 76^\circ \end{array} \rightarrow \\
 3x + 12 + 5x &= 180 \\
 8x + 12 &= 180 \\
 \frac{8x}{8} &= \frac{168}{8} \\
 x &= 21
 \end{aligned}$$

$$\begin{aligned}
 &\begin{array}{c} (4 + 5x)^\circ \\ 4 + 5(29) \\ 149^\circ \end{array} \begin{array}{c} (x + 2)^\circ \\ 29 + 2 \\ 31^\circ \end{array} \\
 4 + 5x + x + 2 &= 180 \\
 6x + 6 &= 180 \\
 6x &= 174 \\
 x &= 29
 \end{aligned}$$

I can solve equations with variables on both sides of the equal sign.

Activity One

Solving
Equations
with Variables
on both Sides
Exploration

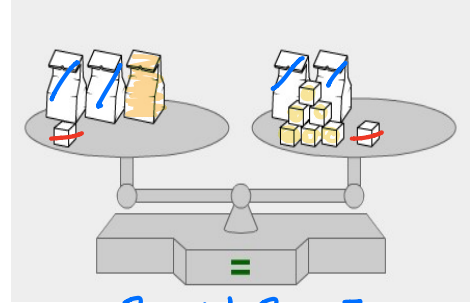
- How many blocks are in one bag?

$$1 \text{ bag} = 6 \text{ blocks}$$

- Write the original problem as an equation, using a variable.

$$3x + 1 = 2x + 7$$

- Solve the equation you wrote algebraically.



$$\begin{array}{r} 3x + 1 = 2x + 7 \\ -2x \quad -2x \\ \hline x + 1 = 7 \\ -1 \quad -1 \\ \hline x = 6 \end{array}$$

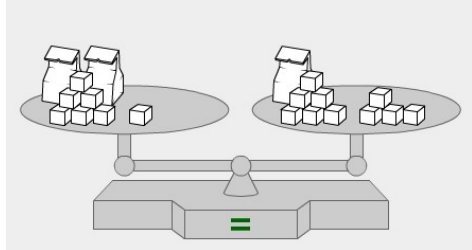
- How many blocks are in one bag?

$$1 \text{ bag} = 3 \text{ blocks}$$

- Write the original problem as an equation, using a variable.

$$2x + 7 = x + 10$$

- Solve the equation you wrote algebraically.



$$\begin{array}{r} 2x + 7 = x + 10 \\ -x \quad -x \\ \hline x + 7 = 10 \\ -7 \quad -7 \\ \hline x = 3 \end{array}$$

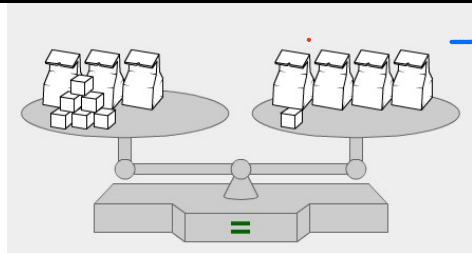
- How many blocks are in one bag?

$$1 \text{ bag} = 5 \text{ blocks}$$

- Write the original problem as an equation, using a variable.

$$3x + 6 = 4x + 1$$

- Solve the equation you wrote algebraically.



$$\begin{array}{r} 3x + 6 = 4x + 1 \\ -3x \quad -3x \\ \hline 6 = x + 1 \\ -1 \quad -1 \\ \hline 5 = x \end{array}$$

SOLVING
MULTI-STEP
EQUATIONS

STEPS:

1. Move all of the variables to the same side (inverse operations)
2. Add or subtract the constant to get the term with the variable alone.
3. Multiply or divide to finish solving.

$$x - 6 = 5x + 10$$

$$-6 = 4x + 10$$

$$-16 = 4x$$

$$-4 = x$$

$$2x - 7 = -5x + 14$$

$$7x - 7 = 14$$

$$7x = 21$$

$$x = 3$$

How are Teddy and Topher's solution strategies the same? How are they different?

both move a variable first


each moves a different variable

Which strategy do you prefer? Why?


Topher's strategy avoids a negative coefficient.

Consider the equation: $5x + 3 = 2x + 5$

Teddy and Topher each solved the equation in a different way. Analyze their solution strategies.

Teddy 

$$\begin{array}{r} 5x + 3 = 2x + 5 \\ -5x \quad -5x \\ \hline 3 = -3x + 5 \\ -5 \quad -5 \\ \hline -2 = -3x \\ -3 = -3 \\ \hline \frac{2}{-3} = \frac{-3x}{-3} \\ \frac{2}{3} = x \\ x = \frac{2}{3} \end{array}$$

Topher 

$$\begin{array}{r} 5x + 3 = 2x + 5 \\ -2x \quad -2x \\ \hline 3x + 3 = 5 \\ -3 \quad -3 \\ \hline 3x = 2 \\ x = \frac{2}{3} \end{array}$$

$$\begin{array}{l} 5x + 15 = 75 - 25x \\ 30x + 15 = 75 \\ 30x = 60 \\ x = 2 \end{array}$$

$$\begin{array}{l} 4x = 20x - 24 \\ -16x = -24 \\ x = \frac{3}{2} \end{array}$$

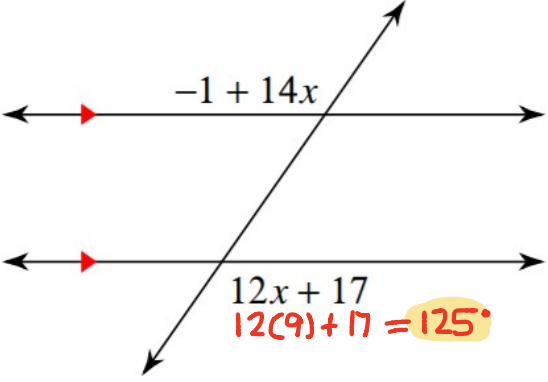
$$\begin{array}{l} -4x - 8 = 2x + 10 \\ -8 = 6x + 10 \\ -18 = 6x \\ -3 = x \end{array}$$

$$\begin{array}{l} -42x = -4x - 1 \\ -38x = -1 \\ x = \frac{1}{38} \end{array}$$

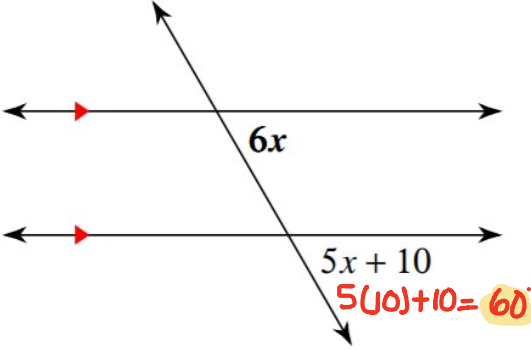
Use multi-step equations to solve for the variable. What is the value of the missing angle?

Application to Angles on a Transversal

DON'T FORGET...
Alternate Interior Angles are Congruent
Corresponding Angles are Congruent



$-1 + 14x = 12x + 17$
 $-1 + 2x = 17$
 $2x = 18$
 $x = 9$



$6x = 5x + 10$
 $x = 10$

SOLVING
Complex
MULTI-
STEP
EQUATIONS

DO I HAVE TO DISTRIBUTE?

$$2+5(x+3)-3(x-4)=x+2(2x+4)$$

Multiply the number outside the parentheses with each term inside.

Combine Like Terms

WARNING: same side of = sign only.

DO I HAVE TO CLEAN UP?

$$2+5x+15-3x+12 = x+4x+8$$

$$2x+9 = 5x+8$$

DO I HAVE VARIABLES ON BOTH SIDES?

$$2x+29 = 5x+8$$

Move the variables to the same side using inverse operations

Move the constant away from the variable using inverse operations

IS THE VARIABLE ISOLATED?

$$29=3x+8$$

IS THE VARIABLE MULTIPLIED?

$$21 = 3x$$

Divide or multiply to get variable by itself.

$$2x-3(x+10)+1=-1+2(x+7)$$

$$2x-3x-30+1=-1+2x+14$$

$$-x-29=2x+13$$

$$-29=2x+13$$

$$-42=2x$$

$$-21=x$$

$$x-6(x-5)=2x+4(x-20)$$

$$x-6x+30=2x+4x-80$$

$$-5x+30=6x-80$$

$$30=11x-80$$

$$110=11x$$

$$10=x$$

$$1+5(7+3x) = 12x+5x$$

$$1+35+15x = 17x$$

$$36+15x = 17x$$

$$36 = 2x$$

$$18 = x$$

$$x+2(2x+3)-1 = \frac{1}{2}(4x+28)$$

$$x+4x+6-1 = 2x+14$$

$$5x+5 = 2x+14$$

$$3x+5 = 14$$

$$3x = 9$$

$$x = 3$$