Name: Key Period:________Date:________________

ALGEBRA 1 SPRING FINAL REVIEW

MODULE 3

Unit 5 – Sequences and Functions

1. Write the first 5 terms of each sequence, then state if it is geometric or arithmetic. How do you know?

a. $f(n) = 2^n$ for $n \ge 0$

$2^0, 2^1, 2^2, 2^3, 2^4, 2^5 \rightarrow 1, 2, 4, 8, 16, 32$ Geometric, $r = 2$

b. $f(n) = 6n + 4$ for $n \ge 0$

$n = 0, 1, 2, 3, 4 \rightarrow 4, 10, 16, 22, 28$ Arithmetic, $d = 6$

c. $f(n+1) = f(n) + 3,\quad f(1) = 4$ for $n \ge 1$

$f(1), f(2), f(3), f(4), f(5) \rightarrow 4, 7, 10, 13, 16, 19$ Arithmetic $d = 3$

d. $a_n = 3^n$ for $n \ge 1$

$n = 1, 2, 3, 4, 5 \rightarrow 3, 9, 27, 81, 243$ Geometric, $r = 3$

2. Given the following sequences, find *f*(2) and state if the sequence is geometric or arithmetic and how you know.

a. 8, 10, 12, 14, 16

$f(2) = 10$ Arithmetic, $d = 2$

b. 3, 6, 12, 24, 48

$f(2) = 6$ Geometric, $r = 2$

c. $f(n) = 4^n$

$f(2) = 16$ Geometric, $r = 4$

d. $f(n) = -2n - 3$

$f(2) = -7$ Arithmetic, $d = -2$

3. What is the difference between a recursive and an explicit formula?

Recursive requires the previous term. (see #1C) Explicit can be used to find any term.

4. Write an **explicit** and a **recursive** formula for the following sequence:

Term:	**	****	******	********
Term #:	1	2	3	4

$f(1) = 2$
$f(2) = 4$
$f(3) = 6$
$f(4) = 8$

$2, 4, 6, 8, \ldots$

Explicit: $f(n) = 2n \quad , n \ge 1$

Recursive: $f(n+1) = f(n) + 2$

Given $f(1) = 2 \quad , n \ge 1$

5. Given $f(x)=\begin{cases} x^2-3 & x\le 2 \\ \dfrac{1}{2}x+7 & x>2 \end{cases}$ evaluate the function at the following x-values:

a. f(0)

$f(0)=0^2-3$
$=-3$

b. f(2)

$f(2)=2^2-3$
$=1$

c. f(6)

$f(6)=\frac{1}{2}(6)+7$
$=10$

6. List the *domain* and *range* of each relation below. Decide if each relation is a function – why or why not?

a. {(1, 2), (3, 4), (5, 6)}

D:{1,3,5}
R:{2,4,6}
Yes

b. {(1, 1), (2, 1), (3, 1)}

D: {1,2,3}
R: {1}
Yes

c.

x	y
3	1
4	2
6	3
3	4
4	5

D:{3,4,6}
R:{1,2,3,4,5}
No

d. relationship between number of hours studied and grade earned on the test D=# of hours R=grades

No, you can get more than one grades for the same amount of hours studied

e. $f(t)=-16t^2+68t+60$ represents the height of a ball thrown vs. time in the air

D: [0,5]
R: [0,132.25]
Yes

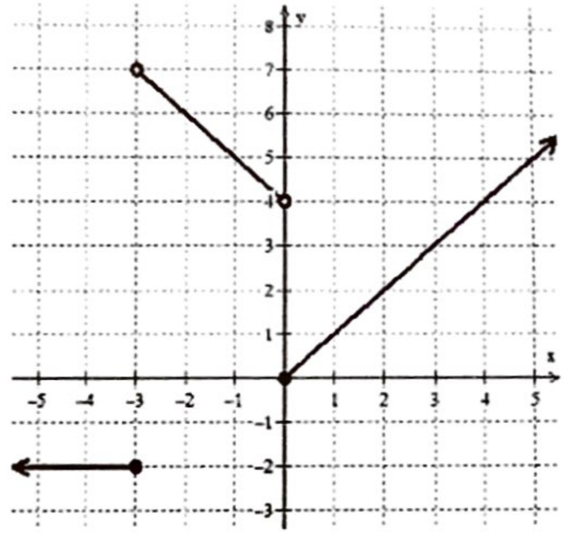
7. Decide if each statement represents a function, **yes** or **no**.

a. The assignment of principals to schools. Yes
b. The assignment of car companies and car models No
c. The assignment of students to their math class (assuming each student takes one math class). Yes
d. The assignment of math classes to a student (assuming each student take one math class). No
e. The assignment of pro-football players to an NFL team. Yes
f. The assignment of NFL teams to their players. No

8. Explain how you know if a graph is a function.

If the graph passes the vertical line test

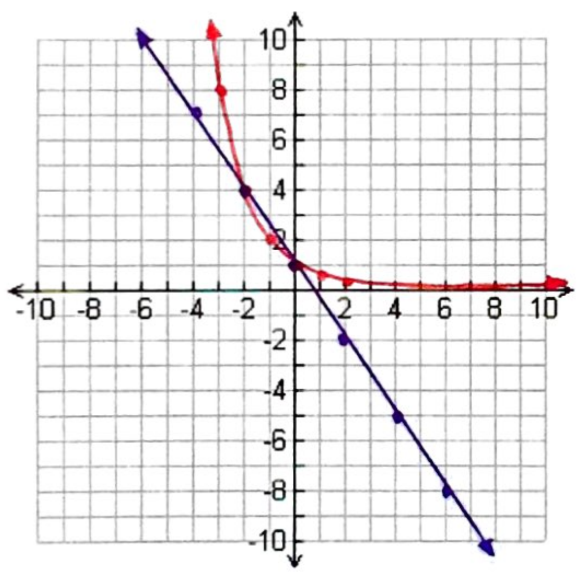
Use the graph to the right to answer # 9-18.



9. Domain $(-\infty,\infty)$
10. Range $-2 \cup [0,\infty)$
11. k(-2) $= 6$
12. k(3) $= 3$
13. k(0) $= 0$
14. k(-8) $= -2$
15. k(-3) $= -2$
16. Interval(s) where increasing $[0,\infty)$
17. Interval(s) where decreasing $(-3,0)$
18. Interval(s) where constant $(-\infty,-3]$

Unit 6 - Exponential Functions

19. On the accompanying grid, sketch the graphs of $f(x)=\left(\frac{1}{2}\right)^x$ and $g(x)=\frac{-3}{2}x+1$. Include several key points on each graph and any other key features. Identify the name of the type (family) of each function and the coordinates of all point(s) of intersection.



$f(x)=\left(\frac{1}{2}\right)^x$

Type of function: exponential

x	f(x)
-2	4
-1	2
0	1
1	1/2
2	1/4

$g(x)=\frac{-3}{2}x+1$

Type of function: linear

x	g(x)
-2	4
0	1
2	-2
4	-5
6	-8

Point(s) of Intersection: (-2,4) (0,1)

20. Label the following as linear decay, linear growth, exponential decay, or exponential growth.

a. Amount of medication remaining in the body over time. exponential decay
b. Population of fish in a pond. exponential growth
c. March Madness playoffs that end in 1 winner. exponential decay
d. Adding $50 to your bank account each month. linear growth

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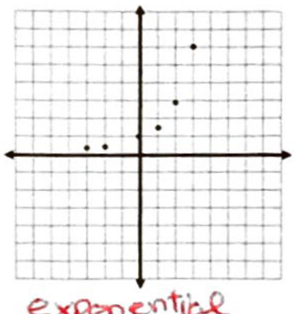
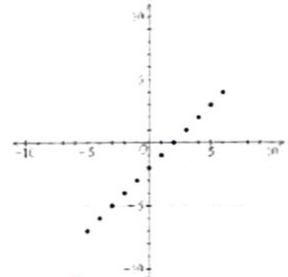
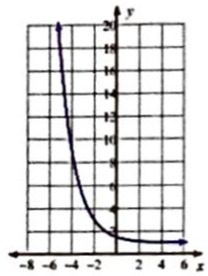


e. exponential growth



f. linear growth



g. exponential decay

x	0	1	2	3
f(x)	-1	3	7	11

h. linear growth

x	0	1	2	3
f(x)	2	6	18	54

i. exponential growth

21. If you are starting a rare stamp collection and you buy 3 new collectable stamps each month, is this situation **linear** or **exponential** growth? Write the equation you would use to find how many stamps you would have after "m" months.

linear growth $f(m) = 3m$

22. If you tell a rumor to 2 people on the first day, and those people each tell 2 more people on the second day, who then tell 2 more people on the 3rd day, does this represent **linear** or **exponential** growth? When will more than 50 people know the rumor not including yourself?

exponential growth $2, 4, 8, 16, 32, 64$

1st 2nd 3rd 4th 5th 6th

$2+4+8+16+32 \rightarrow 62$ people on the fifth day

23. Lori's car value decreases by 25% each year. If she bought the care for \$3000, after how many years will it be worth less than \$1000?

$V(t) = 3000(1-0.25)^t$

$V(t) = 3000(0.75)^t$

$V(4) = 3000(0.75)^4 = 949.22$

After 4 years

24. Identify whether each table contains pairs of values that could be modeled by an **exponential function, linear function, quadratic function**, or **none**.

a.

X	Y
0	1
1	-2
2	-5
3	-8

-3, -3, -3

linear

b.

X	Y
-1	$\frac{1}{4}$
0	$\frac{5}{4}$
1	$\frac{9}{4}$
2	$\frac{13}{4}$

$+\frac{4}{4}$, $+\frac{4}{4}$, $+\frac{4}{4}$

linear

c.

X	Y
-2	0
-1	1
0	0
1	-3

Quadratic

d.

X	Y
-2	$\frac{1}{2}$
-1	1
0	2
1	4

Exponential

Use the exponential growth and decay formulas to answer the following questions.

Exponential growth: $f(t) = A(1+r)^t$ Exponential decay: $f(t) = A(1-r)^t$

25. In 2000 the population of deer in a local forest was approximately 1,100. If the population decreases at a rate of 4%, write an expression and find the deer population **five** years later.

$f(t) = 1100(1-0.04)^t$ $f(5) = 1100(0.96)^5$

$f(t) = 1100(0.96)^t$ $= 896.91$

26. Joe borrows \$500 at 8% interest. Write an **equation** to represent the amount of money *f(t)* that Joe will owe after *t* years.

$f(t) = 500(1+0.08)^t$

$f(t) = 500(1.08)^t$

27. Mary invests \$2000 at .6% interest compounded annually. Write an **equation** to represent the amount of money *f(t)* that Mary will have in the account after *t* years.

$f(t) = 2000(1+0.006)^t$

$f(t) = 2000(1.006)^t$

28. A certain radioactive element decays over time according to the equation $y = A\left(\frac{1}{3}\right)^{\frac{t}{200}}$, where A = the number of grams present initially and t = time in years. If 9000 grams were present initially, how many grams will remain after 400 years?

↳ A

$y = 9000\left(\frac{1}{3}\right)^{400/200}$

$y = 9000\left(\frac{1}{3}\right)^2 = 1000$ grams

29. Which equation models the data in the accompanying table?

Time in hours, x	0	1	2	3	4	5	6
Population, y	5	10	20	40	80	160	320

a. y = 2x + 5 b. $y = 2^x$ c. y=2x d. $y = 5(2)^x$

initial amount, y-intercept growth rate

30. Judy works for a doctor. She placed a sample of bacteria in a culture dish and recorded the number of bacteria present each 30 minutes beginning at 12:00 P.M. The table shows Judy's data. If the pattern of bacterial growth remains constant, how many bacteria should be present in the culture dish at 2:00 P.M.?

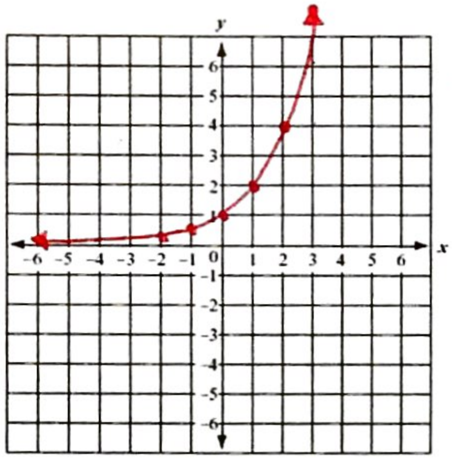
Bacterial Growth		
Time	Number of Bacteria Present	
12:00 P.M.	150	×4
12:30 P.M.	600	×4
1:00 P.M.	2400	
1:30 PM	9600	
2:00 PM	38400	

$f(t) = 150(4)^4$ ← four cycles between 12-2 PM

$= 38400$

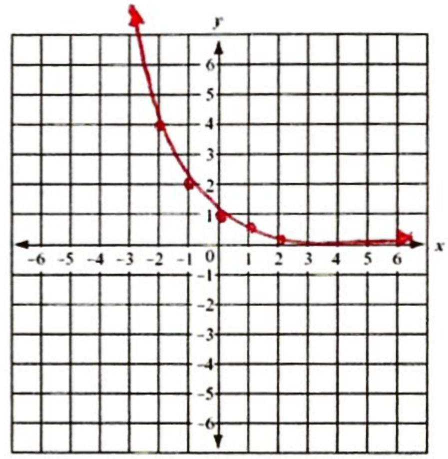
Graph each exponential funtion.

31. $y = 2^x$



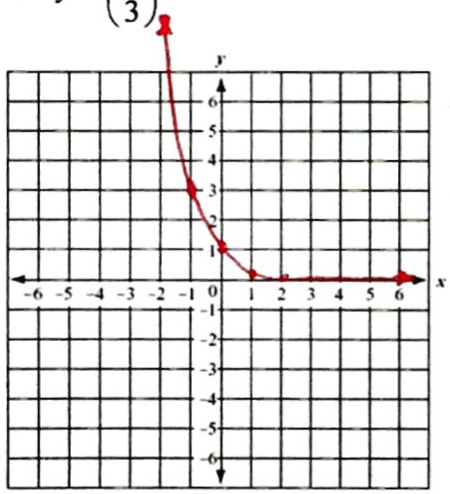
x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

32. $y = \left(\frac{1}{2}\right)^x$



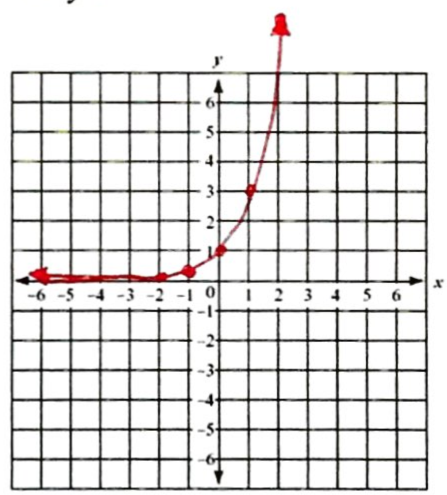
x	y
-2	4
-1	2
0	1
1	1/2
2	1/4

33. $y = \left(\frac{1}{3}\right)^x$



x	y
-2	9
-1	3
0	1
1	1/3
2	1/9

34. $y = 3^x$



x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

35. If the equation $y = 5^x$ is graphed, which of the following values of x would produce a point closest to the x-axis?

a. −3 b. 0 c. 3 d. 5

$5^{-3} = \frac{1}{5^3} = \frac{1}{125} = 0{,}008$

Unit 7 – Transformations of Functions and Modeling

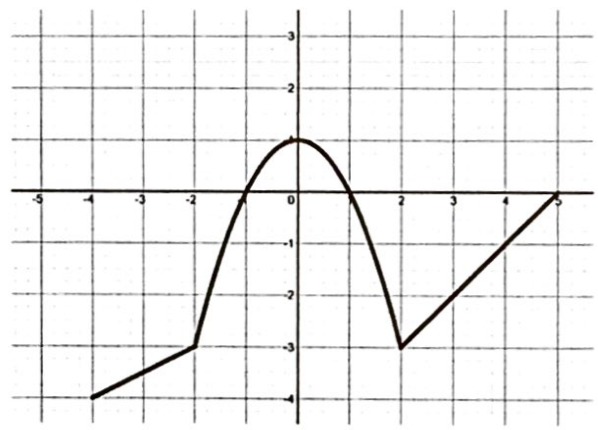
36. Describe the transformations of the absolute value graph described by the equation $k(x) = -\frac{2}{3}|x-3|-4$.

- horizontal shift right 3
- reflect across x-axis
- vertical shrink by 2/3
- vertical shift down 4

37. Write the equation of the absolute value function that stretches (vertically) by a factor of 2, shifts horizontally to the left 5 and vertically up 9.

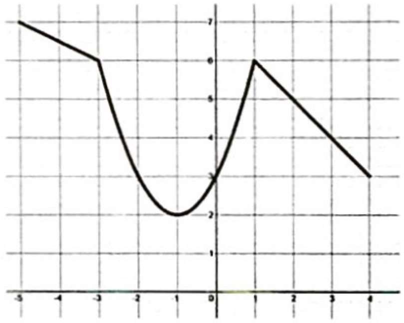
$f(x) = 2|x+5|+9$

#38-40 - The graph of f(x) is given to the right. Use it to match each of the transformations to the appropriate equation below. Describe the transformations shown in each function. One equation will not be used. Justify each answer.



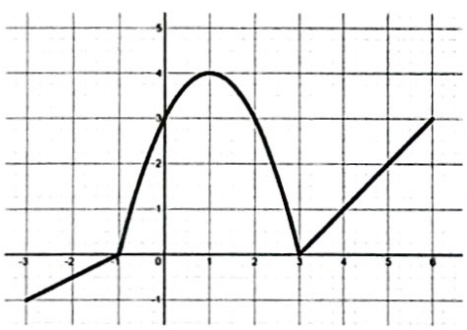
$g(x) = f(x-1)+3$	$h(x) = -f(x+1)+3$
$j(x) = f(x+1)-3$	$k(x) = -f(x-1)+3$

38.



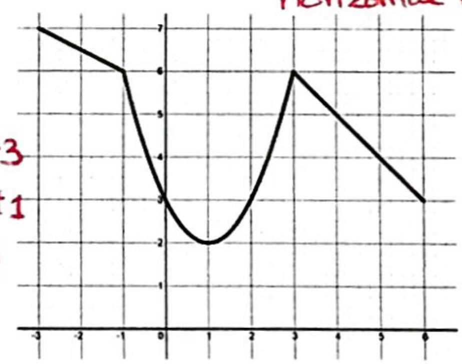
$h(x) = -f(x+1)+3$
reflect x-axis
horizontal left 1
vertical up 3

39.



$g(x) = f(x-1)+3$
horizontal right 1
vertical up 3

40.



$k(x) = -f(x-1)+3$
reflect x-axis
horizontal right 1
vertical up 3

41. If some values of f(x) are shown in the table below, write a new table with the following transformations:

x_1	$f(x_1)$
−1	2
0	0
1	2
2	8

a) g(x) = -f(x) – 3

x	-y	-y-3
-1	-2	-5
0	0	-3
1	-2	-5
2	-8	-11

b) h(x) = f(x-1) +2

x+1	x	y	y+2
0	-1	2	4
1	0	0	2
2	1	2	4
3	2	8	10

MODULE 4

Units 8-10: Quadratics

In #42-50, Factor Completely.

42. $x^2 - 25$

$(x+5)(x-5)$

43. $x^2 - 10x + 24$

$(x-6)(x-4)$

44. $6x^2 - 19x + 8$

48, −16, −3, −19

$6x^2 - 3x - 16x + 8$

$3x(2x-1) - 8(2x-1)$

$(3x-8)(2x-1)$

45. $3x^2 - 18x - 21$

$3(x^2 - 6x - 7)$

$3(x-7)(x+1)$

46. $3xy - 2x^2y + xy^2$

$xy(3 - 2x + y)$

GCF

47. $2x^3y^2 + 6xy^3 + 8xy^2$

$2xy^2(x^2 + 3y + 4)$

GCF

48. $y^2 + 49 + 14y$

$y^2 + 14y + 49$

$(y+7)(y+7)$

$(y+7)^2$

49. $-2k + 3k^2 - 8$

−24, −6, 4, −2

$3k^2 - 2k - 8$

$3k^2 - 6k + 4k - 8$

$3k(k-2) + 4(k-2)$

$(3k+4)(k-2)$

50. $x^3 - 17x^2 + 72x$

72, −9, −8, −17

$x(x^2 - 17x + 72)$

$x(x-9)(x-8)$

51. Given the quadratic equation, $f(x) = x^2 - 4x - 12$, identify the zeros.

$0 = (x-6)(x+2)$

$x = 6$, $x = -2$

52. Given the quadratic equations, write them in **vertex form** by completing the square, then verify that you did it correctly by putting them back into **standard form**.

a. $y = x^2 + 8x + 6$

$y = (x^2 + 8x \underline{+16}) + 6 \underline{-16}$

$y = (x+4)^2 - 10$

b. $f(b) = 2b^2 - 8b + 3$

$f(b) = 2(b^2 - 4b \underline{+4}) + 3 \underline{-8}$

$f(b) = 2(b-2)^2 - 5$

53. Given the quadratic equation $k(x) = 2(x+4)^2 - 9$, identify the translations when compared to the parent graph $y = x^2$.

- vertical stretch by 2
- horizontal shift left 4, vertical shift down 9

54. Find the roots of the following quadratic equation, $(x+2)^2 = 36$.

$\sqrt{(x+2)^2} = \pm\sqrt{36}$

$x+2 = \pm 6$

$x = -2 \pm 6$ → $-2+6 = 4$, $-2-6 = -8$

55. Given the quadratic equation, $y = (x-7)^2 + 8$, identify the Axis of Symmetry and Vertex.

vertex: $(7, 8)$

Axis of sym: $x = 7$

56. Use any method to find the vertex and solutions/roots/zeros. $y = -x^2 - 6x + 16$.

$0 = -1(x^2+6x-16)$

$0 = x^2+6x-16$

$0 = (x+8)(x-2)$

zeros → $x = -8$, $x = 2$

$y = -(x^2+6x+9)+16+9$

$y = -(x+3)^2+25$

vertex: $(-3, 25)$

57. Solve $4(x+2)^2 = 20$.

$\sqrt{(x+2)^2} = \pm\sqrt{5}$

$x+2 = \pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$

58. Find the vertex of the following equations, then state how many solutions/roots/x-intercepts they would each have – how do you know?

a) f(x) = (x-3)(x+2) $x = 3, -2$

$f(x) = x^2 - x - 6$

$x = \frac{3+(-2)}{2} = \frac{1}{2}$

$f(\frac{1}{2}) = (\frac{1}{2})^2 - \frac{1}{2} - 6$

$= \frac{1}{4} - \frac{1}{2} - 6 = \frac{-25}{4}$

Vertex: $(\frac{1}{2}, \frac{-25}{4})$

Two roots

b) $g(x) = x^2 - 3x + 8$ $a=1, b=-3, c=8$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(8)}}{2(1)}$

$x = \frac{3 \pm \sqrt{9-32}}{2}$

$x = \frac{3 \pm \sqrt{-23}}{2}$ No real roots

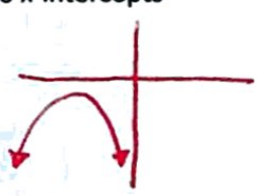
Vertex: $(\frac{3}{2}, \frac{23}{4})$

59. Sketch a graph of a quadratic that has:

a) 2 x-intercepts

b) 1 x-intercept

c) 0 x-intercepts



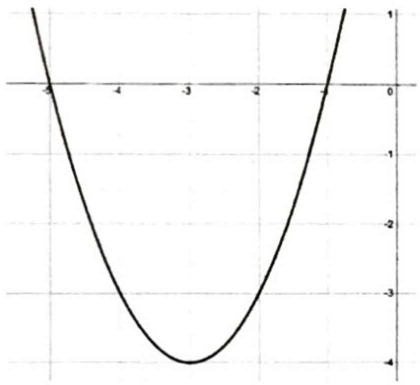
$a=2 \quad b=-4 \quad c=1$

60. How many x-intercepts does $g(x)=2x^2-4x+1$ have? How do you know?

$b^2-4ac \rightarrow (-4)^2-4(2)(1) = 16-8 = 8 > 0$

Two x-intercepts

61. Given the graph, write the function in **vertex form** of the following translation compared to the parent function $y = x^2$.



vertex: $(-3,-4)$

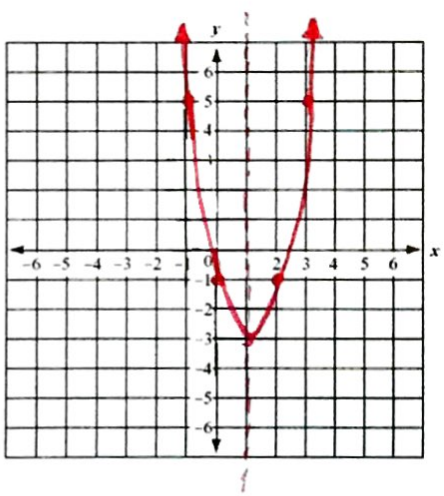
$y = a(x-h)^2+k$

$y = a(x+3)^2-4$

$a=1 \rightarrow$ use $(-2,-3)$

$y = (x+3)^2-4$

62. Graph the quadratic equation$y = 2x^2-4x-1$?



y-intercept: -1

concave up

$x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = 1$

$f(1) = 2(1)^2-4(1)-1$

$= 2-4-1$

$= -3$

vertex: $(1,-3)$

$f(-1) = 2(-1)^2-4(-1)-1$

$= 2+4-1$

$= 6-1$

$= 5$

$(-1,5)$

63. Write a Quadratic Equation that would show a transformation from the parent graph by being vertically shrunk by $\frac{1}{5}$, shifted 7 to the right, and down 10.

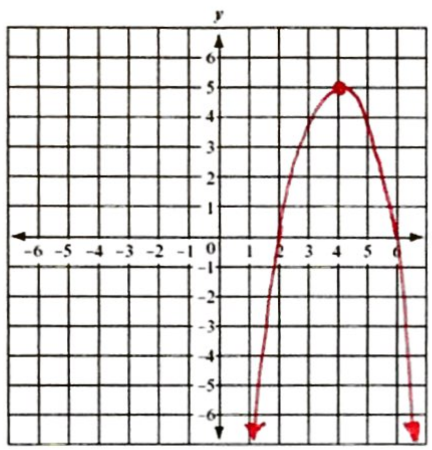
$f(x) = \frac{1}{5}(x-7)^2-10$

64. Write a Quadratic Equation that would show a transformation from the parent graph by being vertically stretched by 4, shifted 2 to the left, and up 3.

$f(x) = 4(x+2)^2+3$

65. Sketch a parabola that shows a vertex of (4,5)

sample answer



66. Sketch the parabola $y = x^2 + 6x + 8$

$0 = (x+4)(x+2)$

$x = -4,\ x = -2$

y-inter: 8

$y = (-3)^2 + 6(-3) + 8$

$= 9 - 18 + 8$

$= -1$

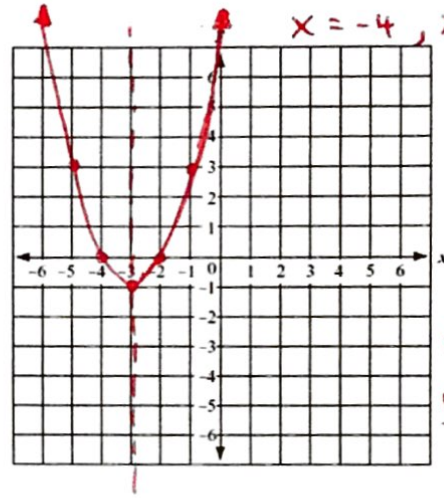
* $(-3, -1)$ vertex

$y = (-1)^2 + 6(-1) + 8$

$= 1 - 6 + 8$

$= 3$

$(-1, 3)$



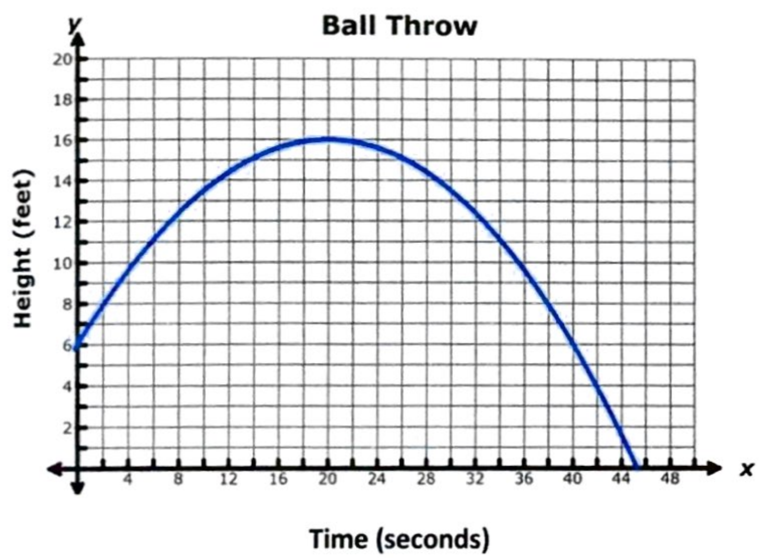
67. Describe the transformation from the parent graph of the following Quadratic Equation: $y = 2(x-4)^2 + 5$

vertical stretch by 2

horizontal shift right 4

vertical shift up 5

68. Given the graph of the quadratic function below, answer the questions:



a. What is the domain of the function graphed? [0, 45]

b. What is the range of the function graphed? [0, 16]

c. At what time does the ball hit its maximum? At 20 seconds

d. What is the maximum height that the ball reaches? 16 feet

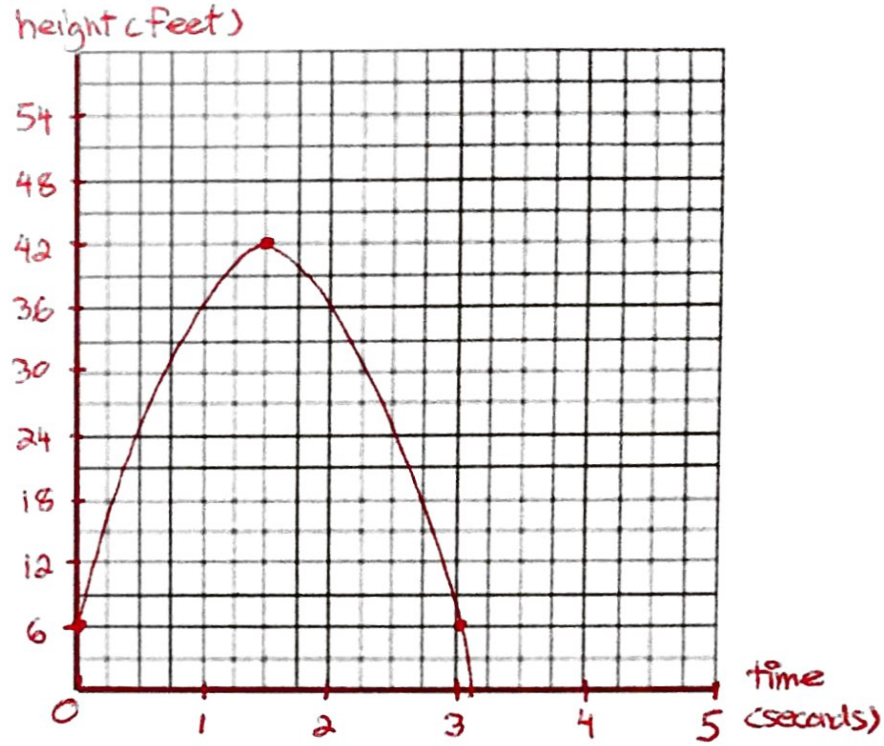
e. If the graph continued to the left, where would the other zero be? -5

f. Where is the y-intercept? 6

g. Where is the graph decreasing? At what interval (20, 45)

h. When is the height of the ball about 11 feet? 6 and 34 seconds

69. Zach throws a hockey puck in the air with an initial velocity of 48 ft/sec from an initial height of 6 feet. **The** quadratic model that represents this situation is $f(t) = -16t^2 + 48t + 6$.



a. When does the puck hit its maximum height?

$t = \frac{-48}{2(-16)} = \frac{-48}{-32} = 1.5$

b. What is the maximum height of the puck?

$f(1.5) = -16(1.5)^2 + 48(1.5) + 6$
$= 42$

c. When does the puck hit the ground?

$t = \frac{-48 \pm \sqrt{(48)^2 - 4(-16)(6)}}{2(-16)}$

$t = \frac{-48 \pm \sqrt{2688}}{-32}$

$t = -0.12$, $\boxed{t = 3.12}$

d. What is the y-intercept? What does it represent?

6 feet is the initial height.

e. Graph the function.

MODULE 4+

Complex Numbers

In # 70-78 simplify completely.

70. $\sqrt{-49} \bullet \sqrt{81}$

$7i \cdot 9 = 63i$

71. i^{175}

$i^{172} \cdot i^3$

$1(i^3) = -i$

72. i^{202}

$i^{200} \cdot i^2$

$1 \cdot i^2 = -1$

73. $(7 + 2i) - (4 + 3i)$

$7 + 2i - 4 - 3i$

$3 - i$

74. $(-3 - 5i) + (2 - 8i)$

$-3 - 5i + 2 - 8i$

$-1 - 13i$

75. $(5 - 6i)(-8 + 9i)$

$-40 + 45i + 48i - 54i^2$

$-40 + 93i + 54$

$14 + 93i$

76. $(5-7i)(5+7i)$

$25+35i-35i-49i^2$

$25+49$

74

77. $\frac{3+8i}{1-4i} \cdot \frac{1+4i}{1+4i}$

$\frac{3+12i+8i+32i^2}{1+16}$

$\frac{-29+20i}{17}$

78. $\frac{-2+5i}{3-7i} \cdot \frac{3+7i}{3+7i}$

$\frac{-6-14i+15i+35i^2}{9+49}$

$\frac{-41+i}{58}$

In # 79-83 plot each point on the coordinate grid.

79. $-1+3i$

80. $-5i$

81. $2-4i$

82. 4

83. $-6-4i$